PAUL CUFFEE SCHOOL

A Maritime Charter School for Providence Youth

9^{th} Grade Mathematics Curriculum

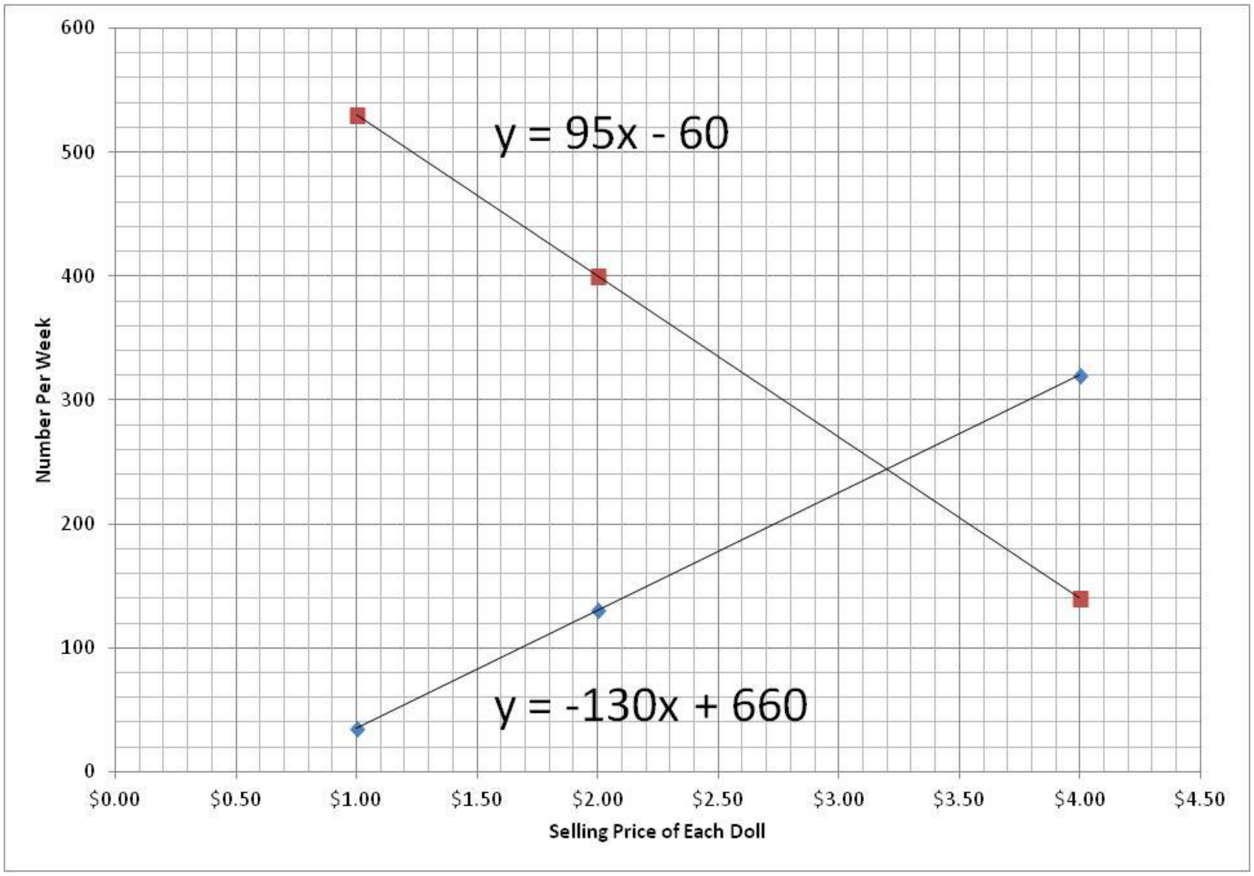
Captain Thomas R. Beall, U. S. Navy (Ret.)

Volume II

Second Trimester

PAUL CUFFEE SCHOOL

A Maritime Charter School for Providence Youth



Systems of Equations

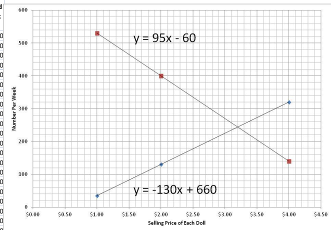
9th Grade Mathematics

Captain Thomas R. Beall, U. S. Navy (Ret.)

Slide 1

Systems of Equations
Solving by Graphing

Selling Price of Each Doll	Number Supplied Per Week Per Store	Number Requested Per Week Per Store
\$1.00	35.00	530.00
\$1.10	44.50	517.00
\$1.20	54.00	504.00
\$1.30	63.50	491.00
\$1.40	73.00	478.00
\$1.50	82.50	465.00
\$1.60	92.00	452.00
\$1.70	101.50	439.00
\$1.80	111.00	426.00
\$1.90	120.50	413.00
\$2.00	130.00	400.00
\$2.10	139.50	387.00
\$2.20	149.00	374.00
\$2.30	158.50	361.00
\$2.40	168.00	348.00
\$2.50	177.50	335.00
\$2.60	187.00	322.00
\$2.70	196.50	309.00
\$2.80	206.00	296.00
\$2.90	215.50	283.00
\$3.00	225.00	270.00
\$3.10	234.50	257.00
\$3.20	**244.00**	**244.00**
\$3.30	253.50	231.00



How else can we find the solution?

Slide 2

Systems of Equations
Solving by Substitution

$y = 95x - 60$

$y = -130x + 660$

$95x - 60 = y = -130x + 660$

$95x - 60 = -130x + 660$

$$
\begin{aligned}
95x - 60 &= -130x + 660 \\
\underline{+130x} \quad & \quad \underline{+130x} \\
225x - 60 &= 660 \\
\underline{+60} \quad & \quad \underline{+60} \\
225x &= 720 \\
\frac{225x}{225} &= \frac{720}{225}
\end{aligned}
$$

$x = 3.2 = \$3.20$

$y = 95(3.2) - 60 = 244$

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Writing the Equations

Example: Mrs. Jones, a math teacher, is celebrating the birthday of the great mathematician Gauss by having a pizza party for her two classes. She orders 3 pizzas and 3 bottles of soda for \$23.34 for her second period class and 4 pizzas and 6 bottles of soda for \$32.70 for her 7th period class. <u>How much does each pizza and bottle of soda cost?</u>

1. Identify the two variables:

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Writing the Equations

Example: Mrs. Jones, a math teacher, is celebrating the birthday of the great mathematician Gauss by having a pizza party for her two classes. She orders 3 pizzas and 3 bottles of soda for $23.34 for her second period class and 4 pizzas and 6 bottles of soda for $32.70 for her 7th period class. How much does each pizza and bottle of soda cost?

2. Write the two equations:

=

=

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Writing the Equations

As a parking attendant, John earns a fixed salary for the first 15 hours he works each week and then additional pay for any time over 15 hours. During the first week, John worked 25 hours and earned $240; the second week, he worked 22.5 hours and earned $213.75. What is John's weekly salary and overtime pay per hour?

1. Identify the two variables:

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Writing the Equations

As a parking attendant, John earns a fixed salary for the first 15 hours he works each week and then additional pay for any time over 15 hours. During the first week, John worked 25 hours and earned $240; the second week, he worked 22.5 hours and earned $213.75. What is John's weekly salary and overtime pay per hour?

2. Write the two equations:

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Systems of Equations
Strategies for Solving

1. *Identifying the variables.*

As a parking attendant, John earns a fixed salary for the first 15 hours he works each week and then additional pay for any time over 15 hours. During the first week, John worked 25 hours and earned $240; the second week, he worked 22.5 hours and earned $213.75. What is John's weekly salary and overtime pay per hour?

The variables are the answers to the question, "What do we need to find out?"

We need to find out what is John's weekly salary and overtime pay per hour.

- ***Let s = John's weekly salary.***
- ***Let o = John's overtime pay per hour.***

2. *Writing the equations.*
3. *Solving the equations.*

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Systems of Equations
Strategies for Solving

1. *Identifying the variables.*

Shopping at Super Sale Days, Martha buys her children 3 shirts and 2 pairs of pants for $85.50. She returns during the sale and buys 4 more shirts and 3 more pairs of pants for $123. What is the sale price of the shirts and pants?

We need to find out what is what is the sale price of the shirts and the sale price of the pants.

- ***Let s = the sale price of the shirts.***
- ***Let p = the sale price of the pants.***

2. *Writing the equations.*
3. *Solving the equations.*

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Systems of Equations
Strategies for Solving

1. *Identifying the variables.*
2. *Writing the equations.*

Shopping at Super Sale Days, Martha buys her children 3 shirts and 2 pairs of pants for $85.50. She returns during the sale and buys 4 more shirts and 3 more pairs of pants for $123. What is the sale price of the shirts and pants?

The equations should include the variables you have identified.

- ***Let s = the sale price of the shirts.***
- ***Let p = the sale price of the pants.***

They should also include any numbers you find in the problem.

- ***3s + 2p = $85.50***
- ***4s + 3p = $123.00***

3. *Solving the equations.*

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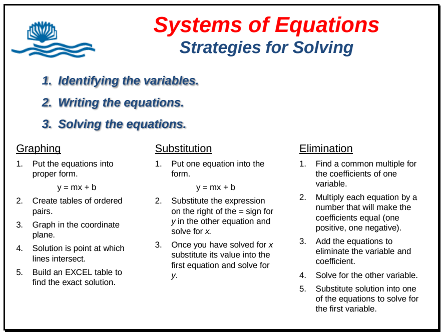

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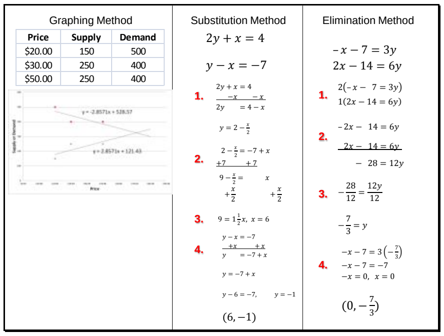

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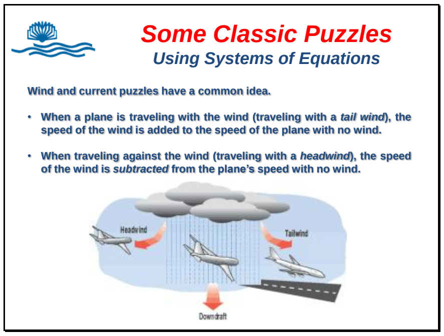

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Airplane Wind Problem

An airplane leaves New York City and heads for Chicago which is 750 miles away. The plane, flying against the wind takes 2.5 hours to reach Chicago. After refueling, the plane returns to New York City, traveling with the wind in 2 hours.

Find the speed of the wind and the speed of the plane with no wind.

1. **Define the variables.**

   Let x = the speed of the plane with no wind.

   Let y = the speed of the wind.

2. **Write the equations.**

   Hint 1: Recall that time x speed = distance.

   Hint 2: Each equation has to include both the wind speed and the plane speed with no wind.

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Airplane Wind Problem

1. **Define the variables.**

   Let x = the speed of the plane with no wind.

   Let y = the speed of the wind.

2. **Write the equations.**

   Hint 1: Recall that time x speed = distance.

   Hint 2: Each equation has to include both the wind speed and the plane speed with no wind.

   $2.5(x-y)=750 \quad 2.5x-2.5y=750$

   $2(x+y)=750 \quad 2x+2y=750$

3. **Solve the equations.**

   338 m.p.h., 38 m.p.h.

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Chemical Solution

Chemical solution problems involve mixing solutions, typically acids, of different strengths. The strengths of the solutions are expressed as percents. The strategy is to keep track of the total amount of acid, in ounces or grams, in the original solution and final solution.

Problem: A 1.5% (of acid) solution is mixed with a 4% solution. How many ounces of each solution are needed to obtain 60 ounces of a 2.5% solution?

1. **Define the variables.**

   Let x = the number of ounces of the 1.5% solution.

   Let y = the number of ounces of the 4% solution.

2. **Write the equations (it might help to make a table).**

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Chemical Solution

	First Solution	Second Solution	Third Solution
Amount of Solution	x	y	60 ounces
Percent Acid			
Amount of Acid			

Equations are:

Solution to the problem:

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Chemical Solution

	First Solution	Second Solution	Third Solution
Amount of Solution	x	y	60 ounces
Percent Acid	1.5%	4%	2.5%
Amount of Acid	0.015x	0.04y	0.025*60

Equations are:

$x + y = 60\ ounces$

$0.015x + 0.04y = (0.025 \times 60) = 1.5\ ounces$

Solution to the problem:

36 ounces of the 4% and 24 ounces of the 1.5%.

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Recall from the lab last week...

We learned how to solve systems of equations using EXCEL:

Example	**A**			**B**		**A INV**			**x = AINV*B**		**CHECK**
	3	3		$23.40		1.00	-0.50		$7.05		$23.40
	4	6		$32.70		-0.67	0.50		$0.75		$32.70
	DET(A) =	6									
Problem 1	**A**			**B**		**A INV**			**x = AINV*B**		**CHECK**
	1	10		$240.00		-3	4		$135.00		$240.00
	1	7.5		$213.75		0.4	-0.4		$10.50		$213.75
	DET(A) =	-2.5									

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Matrix Algebra

A matrix can be formed from the coefficients of two or more linear equations:

$$3p + 3s = \$23.40$$
$$4p + 6s = \$32.70$$

$$A = \begin{bmatrix} 3 & 3 \\ 4 & 6 \end{bmatrix}$$

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Matrix Algebra

Form the matrix for two problems in your project:

$$s + 10o = \$240$$
$$s + 7.5o = \$213.75$$

$A = [\quad]$

$$b + s = \$6000$$
$$0.05b + 0.09s = \$380$$

$A = [\quad]$

Slide 21

Matrix Algebra

Now do it for the remaining 5 problems:

$$s + c = 25$$
$$\$6.99s + \$10.99c = \$230.75$$

$A = [\quad]$

$$2n + 4m = \$205$$
$$3n + 8m = \$342.50$$

$A = [\quad]$

$$3s + 2p = \$85.50$$
$$4s + 3p = \$123$$

$A = [\quad]$

$$1s + 1d = 25$$
$$\$5.99s + \$9.99d = \$189.75$$

$A = [\quad]$

$$1s + 1n = 42$$
$$\$0.20s + \$0.30n = \$11.00$$

$A = [\quad]$

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Matrix Algebra

The inverse of a matrix can be found as follows:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

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To find the inverse of a matrix of the following system of equations:

$s + 10o = \$240$
$s + 7.5o = \$213.75$

Slide 24

Homework

Calculate the determinant for the remaining 5 problems.

Save the matrices you wrote down today. We will need those and your homework for the work we are doing tomorrow.

Slide 25

Matrix Algebra

Now do it for the remaining 5 problems:

$s + c = 25$
$\$6.99s + \$10.99c = \$230.75$

$A = \begin{bmatrix} 1 & 1 \\ \$6.99 & \$10.99 \end{bmatrix}$

$1s + 1d = 25$
$\$5.99s + \$9.99d = \$189.75$

$A = \begin{bmatrix} 1 & 1 \\ \$5.99 & \$9.99 \end{bmatrix}$

$2n + 4m = \$205$
$3n + 8m = \$342.50$

$A = \begin{bmatrix} 2 & 4 \\ 3 & 8 \end{bmatrix}$

$1s + 1n = 42$
$\$0.20s + \$0.30n = \$11.00$

$A = \begin{bmatrix} 1 & 1 \\ \$0.20 & \$0.30 \end{bmatrix}$

$3s + 2p = \$85.50$
$4s + 3p = \$123$

$A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$

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To find the inverse of a matrix of the following system of equations:

$s + 10o = \$240$
$s + 7.5o = \$213.75$

1. Write down the matrix:

$A = \begin{bmatrix} 1 & 10 \\ 1 & 7.5 \end{bmatrix}$

2. Calculate its determinant:

$\frac{1}{|A|} = \frac{1}{ad - bc} = \frac{1}{1 * 7.5 - 1 * 10} = \frac{1}{7.5 - 10} = -\left(\frac{1}{2.5}\right) = -0.4$

3. Calculate the inverse:

$A^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = -0.4 \begin{bmatrix} 7.5 & -10 \\ -1 & 1 \end{bmatrix}$

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Matrix Algebra

$A^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

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Matrix Algebra

$$A^{-1} = \frac{1}{(ad - bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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Matrix Algebra

Homework: Compute the inverse of A for all 5 problems.

$s + c = 25$
$\$6.99s + \$10.99c = \$230.75$

$A = \begin{bmatrix} 1 & 1 \\ \$6.99 & \$10.99 \end{bmatrix}$ $\frac{1}{|A|} = 0.25$

$1s + 1d = 25$
$\$5.99s + \$9.99d = \$189.75$

$A = \begin{bmatrix} 1 & 1 \\ \$5.99 & \$9.99 \end{bmatrix}$ $\frac{1}{|A|} = 0.25$

$2n + 4m = \$205$
$3n + 8m = \$342.50$

$A = \begin{bmatrix} 2 & 4 \\ 3 & 8 \end{bmatrix}$ $\frac{1}{|A|} = 0.25$

$1s + 1n = 42$
$\$0.20s + \$0.30n = \$11.00$

$A = \begin{bmatrix} 1 & 1 \\ \$0.20 & \$0.30 \end{bmatrix}$ $\frac{1}{|A|} = 10$

$3s + 2p = \$85.50$
$4s + 3p = \$123$

$A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$ $\frac{1}{|A|} = 1$

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Matrix Algebra

$s + c = 25$
$\$6.99s + \$10.99c = \$230.75$

$A = \begin{bmatrix} 1 & 1 \\ \$6.99 & \$10.99 \end{bmatrix}$

$2n + 4m = \$205$
$3n + 8m = \$342.50$

$A = \begin{bmatrix} 2 & 4 \\ 3 & 8 \end{bmatrix}$

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Matrix Algebra

$3s + 2p = \$85.50$
$4s + 3p = \$123$

$A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$

$1s + 1d = 25$
$\$5.99s + \$9.99d = \$189.75$

$A = \begin{bmatrix} 1 & 1 \\ \$5.99 & \$9.99 \end{bmatrix}$

Slide 32

Matrix Algebra

$1s + 1n = 42$
$\$0.20s + \$0.30n = \$11.00$

$A = \begin{bmatrix} 1 & 1 \\ \$0.20 & \$0.30 \end{bmatrix}$

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Matrix Algebra

1. To find the solution, multiply A^{-1} by the solution matrix B.

$A^{-1}B = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} e * i + f * j \\ g * i + h * j \end{bmatrix}$

$s + 10o = \$240$
$s + 7.5o = \$213.75$

$A = \begin{bmatrix} 1 & 10 \\ 1 & 7.5 \end{bmatrix}$

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Matrix Algebra

$b + s = \$6000$
$0.05b + 0.09s = \$380$

$$A = \begin{bmatrix} 1 & 1 \\ 0.05 & 0.09 \end{bmatrix}$$

Slide 35

Matrix Algebra

Homework: Find the solution for 5 problems.

$s + c = 25$
$\$6.99s + \$10.99c = \$230.75$

$$A^{-1}B = \begin{bmatrix} 2.75 & -0.25 \\ -1.75 & 0.25 \end{bmatrix} \begin{bmatrix} 25 \\ \$230.75 \end{bmatrix}$$

$1s + 1d = 25$
$\$5.99s + \$9.99d = \$189.75$

$$A^{-1}B = \begin{bmatrix} 2.50 & -0.25 \\ -1.50 & 0.25 \end{bmatrix} \begin{bmatrix} 25 \\ \$189.75 \end{bmatrix}$$

$2n + 4m = \$205$
$3n + 8m = \$342.50$

$$A^{-1}B = \begin{bmatrix} 2 & -1 \\ -0.75 & 0.5 \end{bmatrix} \begin{bmatrix} \$205 \\ \$342.50 \end{bmatrix}$$

$1s + 1n = 42$
$\$0.20s + \$0.30n = \$11.00$

$$A^{-1}B = \begin{bmatrix} \$3.00 & -10 \\ -\$2.00 & 10 \end{bmatrix} \begin{bmatrix} 42 \\ \$11.00 \end{bmatrix}$$

$3s + 2p = \$85.50$
$4s + 3p = \$123$

$$A^{-1}B = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} \$85.50 \\ \$123 \end{bmatrix}$$

Name: ______________________________

Date: ______________________________

Advisor: ______________________________

Solving Systems of Equations by Substitution

Show all work on separate sheets of paper attached to this one.

Example:

$2x + 2y = 0$ (-2, 2)
$6x + y = -10$

a. Solve for y in second equation: $y = -10 - 6x$
b. Remove the y from the first equation: $2x + 2(\quad\quad) = 0$
c. Substitute the value for y in the first equation: $2x + 2(-10 - 6x) = 0$
d. Simplify: $2x - 20 - 12x = 0$
e. Add 20 to both sides: $2x - 12x = 20$
f. Simplify: $-10x = 20$
g. Solve for x: $\frac{-10x}{-10} = \frac{20}{-10}$
$x = -2$
h. Solve for y in second equation: $y = -10 - 6(-2)$
$y = -10 - (-12)$
$y = -10 + 12$
$y = 2$

Solve the following systems by substitution:

1. $2x + 3y = 4$ (5, -2)
$y - 5x = -27$

2. $x - 3y = 2$ (5, 1)
$3x + y = 16$

3. $y + 4x = 19$ (3, 7)
$y - 2x = 1$

4. $3x - y = 3$ (4, 9)
$-6x + 5y = 21$

5. $7y + 3x = 68$ (4, 8)
$y - 4x = -8$

Name: ______________________________

Class: ______________________________

Date: ______________________________

Systems of Equations Review I

1. **Solving Systems of Equations Using Matrix Algebra.** To solve a system of equations using Matrix Algebra for this system of equations:

$$s + 10o = \$240$$
$$s + 7.5o = \$213.75$$

   a. **Write down the matrix from the coefficients of the variables in the system of equations:**

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ 1 & 7.5 \end{bmatrix}$$

   b. **Calculate the determinant of the matrix:**

$$\frac{1}{|A|} = \frac{1}{ad - bc} = \frac{1}{1 * 7.5 - 1 * 10} = \frac{1}{7.5 - 10} = -\left(\frac{1}{2.5}\right) = -0.4$$

   c. **Calculate the inverse (A^{-1}) of the matrix (remember to reorder the matrix as shown below:**

$$A^{-1} = \frac{1}{(ad - bc)} \begin{bmatrix} \boldsymbol{d} & -\boldsymbol{b} \\ -\boldsymbol{c} & \boldsymbol{a} \end{bmatrix} = -0.4 \begin{bmatrix} 7.5 & -10 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 0.4 & -0.4 \end{bmatrix}$$

d. **To find the solution, multiply A^{-1} by the solution matrix B.**

$$A^{-1}B = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix} = \begin{bmatrix} e * i + f * j \\ g * i + h * j \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -3 & 4 \\ 0.4 & -0.4 \end{bmatrix} \qquad B = \begin{bmatrix} \$240 \\ \$213.75 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -3 & 4 \\ 0.4 & -0.4 \end{bmatrix} \begin{bmatrix} \$240 \\ \$213.75 \end{bmatrix} = \begin{bmatrix} -3 * \$240 + 4 * \$213.75 \\ 0.4 * \$240 - 0.4 * \$213.75 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} s \\ o \end{bmatrix} = \begin{bmatrix} \$135.00 \\ \$10.50 \end{bmatrix}$$

2. **Solving Systems of Equations by the Elimination Method.**

   a. **First example:**

$$3x + 4y = 7$$
$$2x - 4y = 13$$

Add together to “eliminate” the “y” and then solve: (4, -1.25);

   b. **Second example:**

$$2x + 3y = 1$$
$$5x + 7y = 3$$

Multiply the first equation by 5 and the second by -2 in order to produce the opposites that will eliminate each other – *why can we do this?*

$$5(2x) + 5(3y) = 5(1)$$
$$-2(5x) + (-2)(7y) = -2(3)$$

$$10x + 15y = 5$$
$$-10x - 14y = -6$$

$$y = -1, x = 2$$

   c. **Third example:**

$$2x + 3y = 1$$
$$-3x - 4y = 0$$

$$3(2x) + 3(3y) = 3(1)$$
$$2(-3x) - 2(4y) = 2(0)$$

$$6x + 9y = 3$$
$$-6x - 8y = 0$$

$$y = 3, x = -4$$

Name: ______________________________

Class: ______________________________

Date: ______________________________

Systems of Equations Review Problems

Show all work on separate sheets of paper attached to this one. Place answers on this sheet.

1. **Solve the following system of equations using Matrix Algebra.**

$$2x + 3y = 1$$
$$5x + 7y = 3$$

a. Matrix A and matrix B are:

$$A = [\qquad\qquad] \qquad B = [\quad]$$

b. The determinant is:

c. The inverse of matrix A is:

$$A^{-1} = [\qquad\qquad]$$

d. The solution to the system of equations is:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

2. **Solve the following system of equations using the Elimination Method.**

$$4x + 3y = 5$$
$$2x + 9y = 7$$

Name: ______________________________

Class: ______________________________

Date: ______________________________

Systems of Equations Review Problems II

Show all work on separate sheets of paper attached to this one. Place answers on this sheet.

1. **Solve the following system of equations using Matrix Algebra.**

$$5x + 7y = 18$$
$$15x + 14y = 50$$

   a. Matrix *A* and matrix *B* are:

$$A = \begin{bmatrix} \quad & \quad \end{bmatrix} \qquad B = \begin{bmatrix} \quad \end{bmatrix}$$

   b. The determinant is:

   c. The inverse of matrix *A* is:

$$A^{-1} = \begin{bmatrix} \quad & \quad \end{bmatrix}$$

d. The solution to the system of equations is:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

2. **Solve the following system of equations using the Elimination Method.**

$$5x + 7y = 18$$
$$15x + 14y = 50$$

Name: ______________________________

Class: ______________________________

Date: ______________________________

Systems of Equations Review Problems III

Show all work on separate sheets of paper attached to this one. Place answers on this sheet.

1. **Solve the following system of equations using Matrix Algebra.**[1]

$$6x + 8y = 24$$
$$12x + 24y = 60$$

a. Matrix A and matrix B are:

$$A = \begin{bmatrix} \quad & \quad \end{bmatrix} \qquad B = \begin{bmatrix} \quad \end{bmatrix}$$

b. The determinant is:

c. The inverse of matrix A is:

$$A^{-1} = \begin{bmatrix} \quad & \quad \end{bmatrix}$$

---

[1] $x = 2, y = 1.5$

d. The solution to the system of equations is:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

2. **Solve the following system of equations using Matrix Algebra.**[2]

$$7x + 11y = 24$$
$$21x + 30y = 60$$

a. Matrix *A* and matrix *B* are:

$$A = \begin{bmatrix} \qquad\qquad \end{bmatrix} \qquad B = \begin{bmatrix} \quad \end{bmatrix}$$

b. The determinant is:

[2] $x = -2.86, y = 4$

c. The inverse of matrix *A* is:

$$A^{-1} = \begin{bmatrix} \quad \end{bmatrix}$$

d. The solution to the system of equations is:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

3. **Solve the following system of equations using the Elimination Method.**[3]

$$6x + 8y = 24$$
$$12x + 24y = 60$$

4. **Solve the following system of equations using the Substitution Method.**[4]

$$7x + 11y = 24$$

$$21x + 30y = 60$$

---

[3] $x = 2, y = 1.5$
[4] $x = -2.86, y = 4$

Name: ______________________________

Class: ______________________________

Date: ______________________________

Systems of Equations Assessment[5]

Show all work on separate sheets of paper attached to this one. Place answers on this sheet.

1. **Solve the following system of equations using Matrix Algebra.**

$$2x + 7y = 12$$
$$10x + 21y = 30$$

a. Matrix A and matrix B are:

$$A = \begin{bmatrix} \quad & \quad \end{bmatrix} \qquad B = \begin{bmatrix} \quad \end{bmatrix}$$

b. The determinant is:

c. The inverse of matrix A is:

$$A^{-1} = \begin{bmatrix} \quad & \quad \end{bmatrix}$$

[5] $x = -1.5, y = 2.14$

d. The solution to the system of equations is:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

2. **Solve the following system of equations using the Elimination Method.**

$$2x + 7y = 12$$
$$10x + 21y = 30$$

Name: ______________________________

Class: ______________________________

Date: ______________________________

Systems of Equations Assessment[6]

Show all work on separate sheets of paper attached to this one. Place answers on this sheet.

1. **Solve the following system of equations using Matrix Algebra.**

$$2x + 7y = 9$$
$$4x + 8y = 12$$

a. Matrix *A* and matrix *B* are:

$$A = \begin{bmatrix} \quad & \quad \end{bmatrix} \qquad B = \begin{bmatrix} \quad \end{bmatrix}$$

b. The determinant is:

c. The inverse of matrix *A* is:

$$A^{-1} = \begin{bmatrix} \quad & \quad \end{bmatrix}$$

[6] $x = 1, y = 1$

d. The solution to the system of equations is:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

2. **Solve the following system of equations using the Elimination Method.**

$$2x + 7y = 9$$
$$4x + 8y = 12$$

Name: ______________________________

Date: ______________________________

Advisor: ______________________________

Algebra I – Systems of Equations – Elimination and Substitution Assessment

Show all work on separate sheets of paper attached to this one.

1. **Solve the following systems of equations using the substitution method.**

   a. $y = 4x - 2$ x = ________________________
   $3x + 2y = 9$ y = ________________________

   b. $4x + 3y = 5$ x = ________________________
   $2x + 9y = 7$ y = ________________________

2. **Solve the following systems of equations using the elimination method.**

   a. $x + 7y = 9$ x = ________________________
   $-x + 14y = 10$ y = ________________________

   b. $-4x + 5y = -3$ x = ________________________
   $3x + 8y = 9$ y = ________________________

Name: ______________________________

Date: ______________________________

Advisor: ______________________________

Algebra I – Systems of Equations – Elimination and Substitution Assessment (Key)

Show all work on separate sheets of paper attached to this one.

1. **Solve the following systems of equations using the substitution method.**

   a. $y = 4x - 2$ x = 1.18
   $3x + 2y = 9$ y = 2.73

   b. $4x + 3y = 5$ x = 0.8
   $2x + 9y = 7$ y = 0.6

2. **Solve the following systems of equations using the elimination method.**

   a. $x + 7y = 9$ x = 2.67
   $-x + 14y = 10$ y = 0.9

   b. $-4x + 5y = -3$ x = 1.47
   $3x + 8y = 9$ y = 0.57

Name: ______________________________

Date: _______________________________

Advisor: _______________________________

Systems of Equations Project
Show work neatly on these papers and any additional sheets you need.

The following problems portray real-world applications of systems of equations with two equations and two unknowns. In each problem, you will be asked to:

Task	**Date Due**
A. Identify the variables of the equations.	
B. Identify and write the two equations.	
C. Find the values of both variables by solving the system of equations using Microsoft EXCEL. *Attach the EXCEL printout to this package.*	
D. Solve ONE of the problems using the graphing method.	
E. Solve TWO of the problems using the elimination method.	
F. Solve TWO of the problems using the substitution method.	
G. Solve THREE of the problems using the Matrix Algebra method.	

This project will be graded on the following scale:

4 - Proficient and exhibiting work above grade level.
3 - Proficient.
2 - Partially proficient.
1 - Below proficient.

A rubric can be found on the last page, fully explaining grading criteria.

Example: Mrs. Jones, a math teacher, is celebrating the birthday of the great mathematician Gauss by having a pizza party for her two classes. She orders 3 pizzas and 3 bottles of soda for $23.34 for her second period class and 4 pizzas and 6 bottles of soda for $32.70 for her 7th period class. How much does each pizza and bottle of soda cost?

1. Identify the two variables:

   Let *p* equal the price of a pizza.
   Let *s* equal the price of a bottle of soda.

2. Write the two equations:

   a. *3p + 3s = $23.40*
   b. *4p + 6s = $32.70*

3. Solve the system:

   Solve for *p* by substitution:

$$\begin{array}{l} 3p + 3s = \$23.40 \\ \underline{\quad - 3s \qquad\quad - 3s} \end{array}$$

$$3p \quad = \$23.40 - 3s$$

$$\frac{3p}{3} \quad = \frac{\$23.40}{3} - \frac{3s}{3}$$

$$p \quad = \$7.80 \quad - s$$

   Substitute the value of *p* into one of the original equations to solve for *s*.

$$4(\$7.80 - s) + 6s = \$32.70$$

$$\begin{array}{l} \$31.20 - 4s + 6s = \$32.70 \\ \underline{-\$31.20 \qquad\qquad - \$31.20} \end{array}$$

$$-4s + 6s = \$1.50$$
$$2s \quad = \$1.50$$

$$\frac{2s}{2} = \frac{\$1.50}{2}$$

$$s = \$0.75$$

$$p = \$7.80 - (\$0.75) = \$7.05$$

Problem 1: Income

As a parking attendant, John earns a fixed salary for the first 15 hours he works each week and then additional pay for any time over 15 hours. During the first week, John worked 25 hours and earned \$240; the second week, he worked 22.5 hours and earned \$213.75. What is John's weekly salary and overtime pay per hour?

Ans.: John's weekly salary is \$135, plus \$10.50 for overtime.

1. Identify the two variables:

   Let s = the John's weekly salary.
   Let o equal what John is paid for each overtime hour.

2. Write the two equations:

   Since John is paid a fixed amount of money for his first 15 hours, we will first need to find out how many overtime hours he worked each week.

   a. In the first week, he worked 25 total hours. From that you subtract 15 hours (the number of hours for which he gets a fixed salary) to get a total of 10 overtime hours.

   b. In the second week, he worked 22.5 total hours. From that you subtract 15 hours to get t total of 7.5 overtime hours.

   a. $s + 10o = \$240$
   b. $s + 7.5o = \$213.75$

3. Solve the system:

Problem 2: Investments

Mr. Moore sells his tractor for \$6000. He makes two investments, one at a bank paying 5% interest per year and the rest in stocks yielding 9% interest per year. If he earns a total of \$380 in interest in the first year, how much did he invest at the 5% rate and how much at the 9% rate.

Ans.: \$4000 at 5%, \$2000 at 9%.

1. Identify the two variables:

   Let b equal the amount Mr. Moore put in the bank.
   Let s equal the amount Mr. Moore invested in the stock market.

2. Write the two equations:

3. Solve the system:

Problem 3: Business

At a local music store, single - recording tapes cost \$6.99 and concert tapes cost \$10.99 At the end of the day a total of 25 tapes were sold. If the total sales that day were \$230.75 for single - recording tapes and concert tapes, find the number of each type sold.

Ans.: 11 single tapes, 14 concert tapes.

1. Identify the two variables:

2. Write the two equations:

3. Solve the system:

Problem 4: Finance

To rent an apartment, a one - time deposit is frequently required with the first month's rent. Roberto paid \$900 the first month and a total of \$6950 during the first year. Find the monthly rent and the amount of the deposit.

Ans.: rent = \$550, deposit = \$350.

1. Identify the two variables:

2. Write the two equations:

3. Solve the system:

Problem 5: Recreation

The Shamrock Inn offers two holiday weekend specials. The first is 2 nights and 4 meals for \$205, while the other is 3 nights and 8 meals for \$342.50. At these rates, what is the room rate and the cost of a meal?

Ans.: Rooms cost \$67.50 per night and meals cost \$17.50 each.

1. Identify the two variables:

2. Write the two equations:

3. Solve the system:

Problem 6: Consumer Math

Shopping at Super Sale Days, Martha buys her children 3 shirts and 2 pairs of pants for $85.50. She returns during the sale and buys 4 more shirts and 3 more pairs of pants for $123. What is the sale price of the shirts and pants?

Ans.: Shirts cost $10.50 each, and pants cost $27 each.

1. Identify the two variables:

2. Write the two equations:

3. Solve the system:

Problem 7: Business

At a local bakery, single - crust pies sell for $5.99 and double - crust pies sell for $9.99. The total number of pies sold on Monday was 25. If the sales total for Monday was $189.75 for those two types of pies, find the number of each type sold.

Ans.: 15 single - crust pies and 10 double - crust pies were sold.

1. Identify the two variables:

2. Write the two equations:

3. Solve the system:

Problem 8: Sales

The owner of a health - food store sells a mixture of seeds and nuts. The seeds cost 20 cents per ounce, and the nuts cost 30 cents per ounce. 42 ounces of the mixture has a value of $11.00. How many ounces of seeds and nuts did the owner use in the mix?

Ans.: There were 16 ounces of seeds and 26 ounces of nuts in the mixture.

1. Identify the two variables:

2. Write the two equations:

3. Solve the system:

Rubric

	4	3	2	1
Graphing	A complete coordinate system is shown with all required components, including: • Each axis appropriate-ly scaled. • Each axis properly labeled. • All data points labeled with ordered pairs. • Two lines drawn with correct labels.	A coordinate system is shown with most components but missing one of the following: • Each axis appropriate-ly scaled. • Each axis properly labeled. • All data points labeled with ordered pairs. • Two lines drawn with correct labels.	A coordinate system is shown with two lines drawn on correct data points but missing more than one of the following components: • Each axis appropriate-ly scaled. • Each axis properly labeled. • All data points labeled with ordered pairs.	No graph is drawn.
Complete Work Shown	Each step in the method chosen to solve the problem is clearly labeled and shown. Equations and calculations are written neatly and clearly. There are no arithmetic errors. The answer is	Each step in the method chosen to solve the problem is clearly labeled and shown but one of the following is missing: • Equations and calculations are written neatly and clearly.	Each step is not clearly shown or labeled and / or more than one of the following is missing: • Equations and calculations are written neatly and clearly.	No work is shown.

	4	3	2	1
	clearly indicated in an appropriate place.	• There are no arithmetic errors. • The answer is clearly indicated in an appropriate place.	• There are no arithmetic errors. • The answer is clearly indicated in an appropriate place.	
Correct Answer	Answer is complete and correct with all work shown.	Answer is correct but some small details of the work are not shown ***or*** answer is incorrect due to a minor arithmetic error which is clearly recognizable in the work shown ***or*** the answer is correct but some part is left out (such as identifying the variables or writing the equations).	Answer is complete and incorrect but enough work is shown to identify what led to the correct answer so that the student can easily correct it.	Answer is blank or incorrect with inadequate work shown to identify the cause of the error.

PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth

Logic & Proof

9th Grade Mathematics

Captain Thomas R. Beall, U. S. Navy (Ret.)

Grade / Content Area	**9^{th} Grade: Geometry**
Lesson Title	**Transformations / Logic and Proof I**
National and State Content Standards	*State Content Standards*: I. **M(G&M)-10-2: Makes and defends conjectures**, constructs geometric arguments, uses geometric properties or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g. Pythagorean Theorem, Triangle Inequality Theorem). II. **M(G&M)-10-4: Applies the concepts of congruency by solving problems on and off the coordinate plane involving reflections, translations, or rotations;** or solves problems using congruency involving problems within mathematics or across disciplines or contexts.
	Common Core Standards: I. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. II. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. III. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. I. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
Student Learning Objectives	I. Students will state the meaning of terms including 'definition', 'postulate', 'theorem'. II. Students will be able to prove simple geometric concepts using the principles of logic.
Opportunities to Learn	I. *Outline for the Week.*

A. **Day 1:**

1. We will begin our study of proofs by discussing what they are and then doing a number of "proofs without words":

   a. Pythagorean Theorem (http://illuminations.nctm.org/ActivityDetail.aspx?ID=30).

   b. The three challenges on page 81 of Holt, *Geometry*.

2. Students will be asked to write what they have observed.

B. **Day 2:**

1. We will review the previous day's concepts.

2. We will consider the concepts of logic and how they relate to proofs.

   a. We will consider Euler diagrams and how they can illustrate logical ideas.

   b. We will consider important geometry terms: definitions, postulates, theorems, lemmas.

3. Students will then work on homework problems from Holt, *Geometry*, pages 83 – 85.

C. **Day 3:** With the tools from the previous lessons, we will attempt a number of proofs using exercises from Holt, *Geometry*.

D. **Day 4:** If necessary, we will take more time on proofs. We will conclude the week with a quiz.

II. *Classroom Organization*: New classroom organization. We will have students working in pairs or in groups of four.

III. *Differentiation*.

IV. *Materials:*

A. For each group of students:

1. **Day 1:** Template of triangles to cut out and use to prove the Pythagorean Theorem, scissors, worksheet for this activity, copies of pages 80 and 81 of Holt, *Geometry*.

	2. **Day 2:** Copies of Holt, *Geometry*, pages 80 – 81. 3. **Day 3:** Worksheet, copies of Holt, *Geometry*, pages 95 – 96. 4. **Day 4:** Worksheet, Copies of Holt, *Geometry*, page 103. B. Additional materials: 1. None.
Instructional Procedures	I. **Day 1:** A. *Warm-up* (**10 minutes**). B. *Launch* (**20 minutes**). 1. I will state, *"We have considered the building blocks of geometry: points, lines and segments, angles, and their transformations. With these, we can visualize how to construct any shape anywhere in the cosmos. Now we will consider how we can use these building blocks to prove more complex geometric concepts that we will use throughout the remainder of the course.* 2. *"A lot of statements in mathematics we accept as fact without any proof. For instance, consider the sum:* $$2 + 2 = 4$$ *"We accept this as true, almost as something ordained by the universe. And yet, it is possible to imagine a situation in which:* $2 + 2 = 0$ and $2 + 2 = 1$ *"We won't spend any time on this – if you want to explore it further, take a course in abstract algebra when you hit college. The point is that those things we accept as inalienable truths are often not so true after all. In order for something to be useful in mathematics, we need to do more than accept things on face value. We need to subject mathematical ideas to rigorous proof.* 3. **Vocabulary:** a. **Definitions:** i. *Logic:* the science of the formal principles of reasoning.

ii. *Proof:* a convincing argument that uses logic to show that a statement is true.

iii. *Axiom:* in logic, an indemonstrable first principle, rule, or maxim, that has found general acceptance (i.e. it doesn't require a proof to be accepted).

iv. *Theorem:* a proposition or statement that is demonstrated through logical proof (i.e. a proof is required).

v. *Corollary:* a statement that follows readily from a previous statement (normally a theorem).

b. **Euclid's Axioms (building blocks of Euclidean Geometry):**

i. Given two points there is one straight line that joins them.

ii. A straight line segment can be prolonged indefinitely.

iii. A circle can be constructed when a point for its center and a distance for its radius are given.

iv. All right angles are equal.

v. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.

vi. Things equal to the same thing are equal.

vii. If equals are added to equals, the wholes are equal.

viii. If equals are subtracted from equals, the remainders are equal.

ix. Things that coincide with one another are equal.

x. The whole is greater than a part.

C. *Engagement* (**40 minutes**).

1. I will state, *"Let's consider the Pythagorean Theorem. How can we*

prove that it is always true?

$$a^2 + b^2 = c^2$$

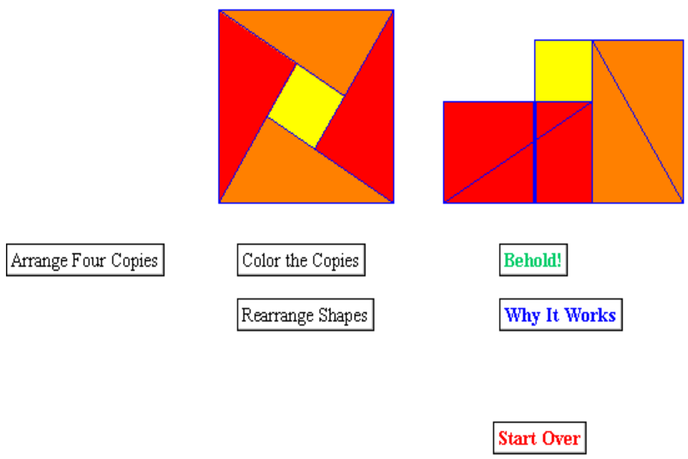
2. First Activity (**20 minutes**): Students will:

    a. Cut out the four triangles and the small square.

    b. Measure the dimensions and record them.

    c. Attempt to arrange the figures as in the diagram below to prove the theorem.

    d. Record their findings and describe the proof in their own words. Groups will be asked to report out findings.



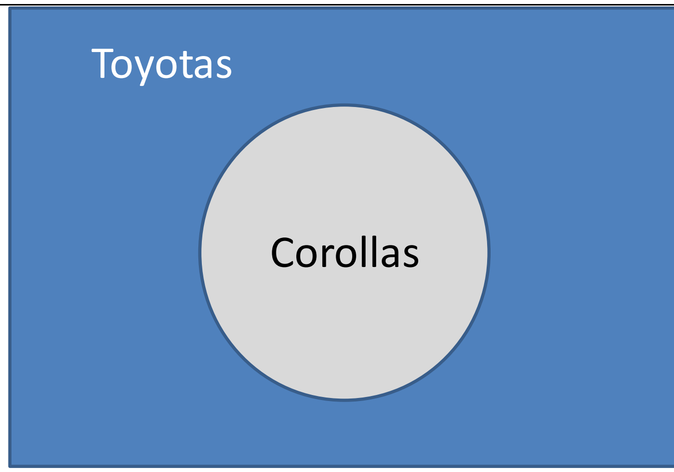
3. Second Activity (**20 minutes**): *"We know that a proof is an argument that uses logic but what is logic?"*

    Answer: "*An argument is an attempt to demonstrate the truth of an assertion called a conclusion, from a hypothesis and a set of true assertions called premises."*

    "Let's look at a simple example."

    If a car is a Corolla then it is a Toyota.

    "This can be illustrated by what is called an Euler Diagram:



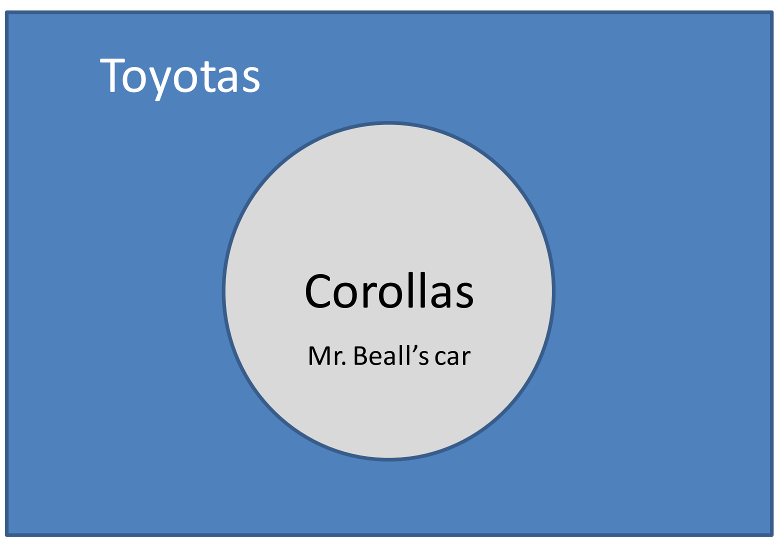
"The hypothesis (labeled p), 'car is a Corolla', *is proven by the conclusion (labeled q)* 'it is a Toyota'. *The entire statement is called a* conditional *and, in its generalized form is:*

If p then q

or

$$p \Rightarrow q$$

"Now consider the following statement 'Mr. Beall's car is a Carolla.' Placing that in the Euler diagram:



"How could we express this?"

Answer:

1. If a car is a Corolla, then it is a Toyota.
2. Mr. Beall's car is a Corolla.
3. Therefore, Mr. Beall's car is a Toyota.

or

Mr. Beall's car is a Toyota, therefore it is a Corolla.

"This is called deductive reasoning."

Definition: Deductive Reasoning: The process of drawing conclusions by using logical reasoning in an argument.

"Now what if I turned this around and said:

Mr. Beall's car is a Toyota, therefore it is a Corolla.

"This is what we call the **converse** *of the original statement. Is it true?*

D. *Closing. "Tomorrow we will expand this discussion into geometrical properties. Finish the assigned problems as homework."*

II. **Day 2:**

A. *Warm-up* (**10 minutes**). Find the product or quotient:

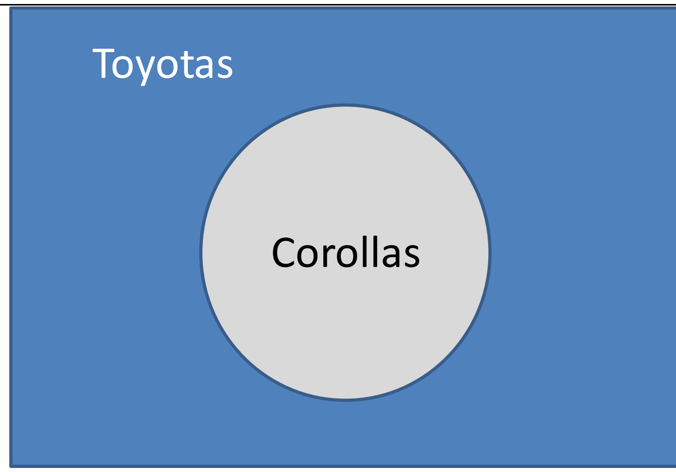
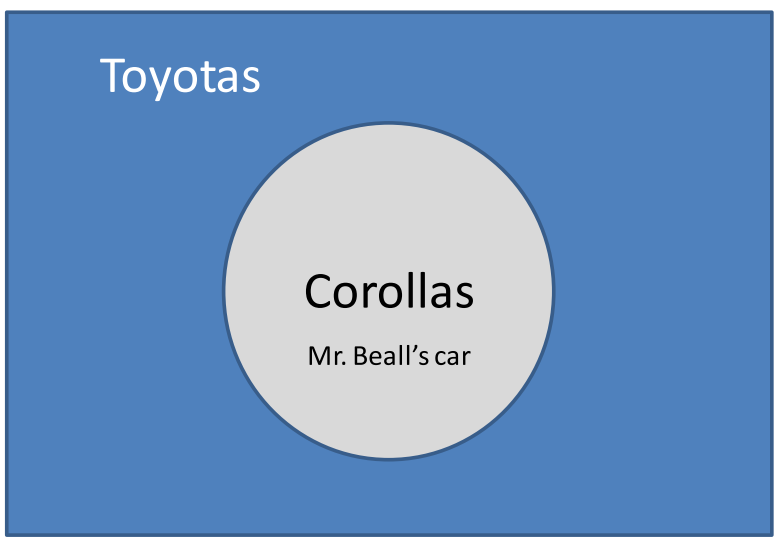
$$(5)(-7), (-12)(-11), (-18) \div (6), (-24) \div (-4), \left(\frac{7}{8}\right)\left(\frac{5}{4}\right), \left(\frac{12}{16}\right) \div (\frac{4}{3})$$

B. *Homework Review* (**20 minutes**):

1. Review conditional statements and Euler diagrams.

If a car is a Corolla then it is a Toyota.

"This can be illustrated by what is called an Euler Diagram:

	 *"The hypothesis (labeled p),* 'car is a Corolla', *is proven by the conclusion (labeled q)* 'it is a Toyota'. *The entire statement is called a* conditional *and, in its generalized form is:* If p then q or $p \Rightarrow q$ *"Now consider the following statement 'Mr. Beall's car is a Carolla.' Placing that in the Euler diagram:*  *"How could we express this?"*

Answer:

a. If a car is a Corolla, then it is a Toyota.
b. Mr. Beall's car is a Corolla.
c. Therefore, Mr. Beall's car is a Toyota.

or

Mr. Beall's car is a Toyota, therefore it is a Corolla.

"This is called deductive reasoning."

Definition: Deductive Reasoning: The process of drawing conclusions by using logical reasoning in an argument.

"Now what if I turned this around and said:

Mr. Beall's car is a Toyota, therefore it is a Corolla.

"We decided this was called the **converse** *of the statement but showed it was not true."*

2. Working in groups, students will compare homework answers and then provide answers to the class. Pay particular attention to 13 – 23:

   a. Nr. 15: If one is a flutist, then one is a musician.

   b. Nr. 19: If the measure of **each angle is less than 90°,** then the **triangle is acute**.

      i. Hypothesis: each angle is less than 90°.
      ii. Conclusion: triangle is acute.

   c. Nr. 23: If $m\angle BXC + m\angle CXC = 90°, then\ m\angle AXB = 90°$.

   d. Nr. 25: Socrates is mortal.

C. *Launch* (**15 minutes**). I will state: *"Now let's consider a statement which is true and also its converse is true:*

1. "A **teenager is a person** from **13 to 19 years old**." A statement like this, written as a statement of fact, is called a definition.

   a. A definition can be written as a conditional.
   b. Its converse is equally true.

2. If someone is 13 to 19 years old then that person is a teenager. *Conditional* $p \Rightarrow q$

3. If someone is a teenager then that person is 13 to 19 years old. *Converse* $q \Rightarrow p$

4. This statement is called **bi-conditional**. $p \Leftrightarrow q$

"Now let's consider a geometric property:"

Definition: Adjacent angles are angles in a plane that have their vertices and one ray in common but do not overlap.

*"**Take two minutes** to restate this as a conditional and also write its converse. Determine whether the statement is bi-conditional."*

Answer:

i. *Conditional:* If and two angles in a plane have their vertices and one ray in common but do not overlap, then those angles are adjacent.

ii. *Converse:* If two angles are adjacent, then they have their vertices and one ray in common but do not overlap.

iii. It is bi-conditional.

"A bi-conditional statement can be written as 'if and only if'. For example:

iv. Two angles in a plane are adjacent if and only if they have their vertices and one ray in common but do not overlap.

D. *Engagement* (**25 minutes**).

	1. First Activity (**15 minutes**): Working in groups, students will answer exercises 8 – 16 on page 103. 2. Second Activity (**10 minutes**): Students will complete a worksheet with using the Algebraic Properties of Equality. I will state, *"Now let's bring this back to mathematics. Copy the following that is on the board into your notes. Notice that these are written as **conditional** statements.* **Algebraic Properties of Equality** (table below) E. *Closing* (**5 minutes**). I will state, *"Try doing this proof over the weekend. Next week we will take up more of these proofs using definitions to prove geometric theorems.*
Assessment	1. Worksheets 2. Homework assignments.

Algebraic Properties of Equality

Addition Property	$if\ a = b, then\ a + c = b + c$
Subtraction Property	$if\ a = b, then\ a - c = b - c$
Multiplication Property	$if\ a = b, then\ ac = bc$
Division Property	$if\ a = b\ and\ c \neq 0, then \frac{a}{c} = \frac{b}{c}$
Substitution Property	If a = b, then you may replace a with b in any true equation containing a and the resulting equation will still be true.

Name: ______________________

Class:______________________

Date:______________________

Proof Without Words – Pythagorean Theorem

1. Cut a right triangle from the sheet of paper. The sides of the triangle should be:

$$a = 3\ in., b = 4\ in., c = 5\ in.$$

2. Trace and cut out three additional congruent triangles.

3. Using different color paper, cut out a small square of dimensions 1 inch by 1 inch.

4. Measure the dimensions of each right triangle and record them:

   a. Side a = ______________________

   b. Side b = ______________________

   c. Side c = ______________________

   d. Small square = ________________

5. Arrange and then rearrange the figures to prove the Pythagorean Theorem. What you want to show is that:

$$a^2 + b^2 = c^2$$

6. Record your findings and state your proof in your own words:

Name: ___________________________

Class:___________________________

Date:___________________________

Logic – Euler Diagrams

1. Recall the following definitions:

   An equilateral triangle is a triangle with three congruent sides.
   An isosceles triangle is a triangle with at least two congruent sides.

   a. Draw and Euler diagram that conveys the following information:

      i. If a triangle is equilateral, then the triangle is isosceles.
      ii. Triangle ΔABC is equilateral.

   b. What conclusion can you draw about ΔABC?

2. Write the **converse** of the statement, "If a triangle is equilateral, then the triangle is isosceles."

3. Is this converse statement true? Why or why not?

4. Consider the following statements:

All United States postal workers are federal employees.
John is a United States postal worker.

a. Write the first statement as a **conditional.**

b. Draw an Euler diagram to illustrate the relationship between the two statements.

c. Use **deductive reasoning** to show that John is a federal employee.

Worksheet – Page 1

Algebraic Properties of Equality

Addition Property	$if\ a = b, then\ a + c = b + c$
Subtraction Property	$if\ a = b, then\ a - c = b - c$
Multiplication Property	$if\ a = b, then\ ac = bc$
Division Property	$if\ a = b\ and\ c \neq 0, then \frac{a}{c} = \frac{b}{c}$
Substitution Property	If $a = b$, then if you replace a with b in any true equation containing a then the resulting equation will still be true.

1. Write the converse of each statement and indicate if the converse is true.

2. Given the following number line:

a. $AB = CD$.

b. Use the Algebraic Properties of Equality to prove that AC = BD.

<table>
<tr><td>Grade / Content Area</td><td>9th Grade: Geometry</td></tr>
<tr><td>Lesson Title</td><td>Proofs II</td></tr>
<tr><td rowspan="2">National and State Content Standards</td><td>State Content Standards:

I. M(G&M)-10-2: Makes and defends conjectures, constructs geometric arguments, uses geometric properties or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g. Pythagorean Theorem, Triangle Inequality Theorem).</td></tr>
<tr><td>Common Core Standards:

I. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.</td></tr>
<tr><td>Student Learning Objectives</td><td>I. Students will state the meaning of terms including 'definition', 'postulate', 'theorem'.

II. Students will be able to prove simple geometric concepts using the principles of logic.</td></tr>
<tr><td>Opportunities to Learn</td><td>I. Outline for the Week.

A. Day 1: We will review the concepts that we covered in the past week. I will provide a note-taking sheet to ensure we cover all definitions and concepts. We will then complete a worksheet that includes the Algebraic Properties of Equality (to be used in our first simple formal proof). These concepts will include:

1. Definition of a proof.
2. Description of an Euler diagram.
3. Definition of a conditional statement.
4. Components of a conditional statement (hypothesis and conclusion).
5. Definition of a converse of a conditional statement.
6. Definition of a bi-conditional statement.

B. Day 2:

• I will start by demonstrating a simple "two column proof" using the</td></tr>
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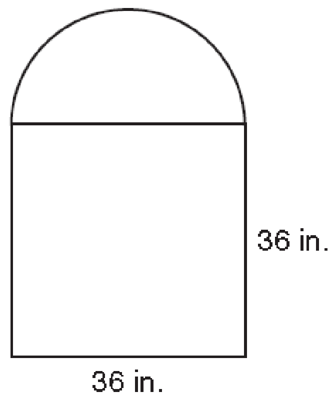
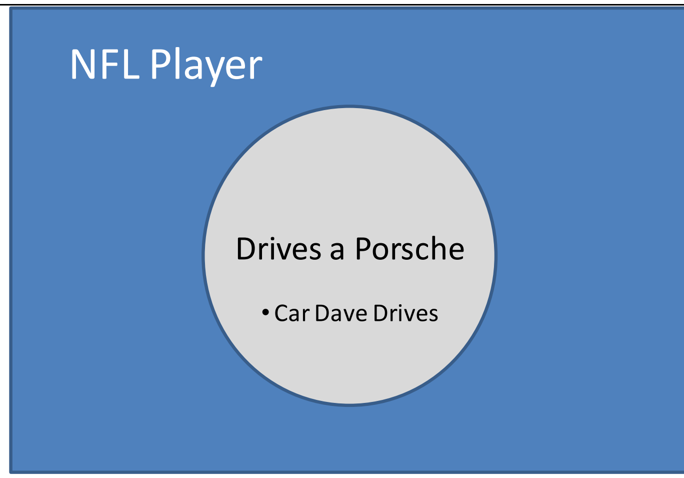
	Algebraic Properties of Equality from page 109 of Holt, *Geometry*. • Students will then begin a worksheet that includes problems 9-12 from page 112 of Holt, *Geometry*. • We will then review a number of Theorems and Properties (provided in a notetaking guide): - Overlapping Segments Theorem. - Overlapping Angles Theorem. - Equivalence Properties of Equality. - Equivalence Properties of Congruence. • Students will then work in groups to do a number of problems from Holt, *Geometry* to be completed as homework. C. **Day 3:** We will continue with simple two column proofs, performing a number of exercises in class. In the process we will introduce the relationship between theorems and proofs. D. **Day 4:** We will continue practice with proofs – using manipulatives from the *Illuminations* lesson Pieces of Proof. E. **Day 5:** We will complete the week with a quiz. II. *Classroom Organization*: Students will work in pairs or groups of 3 – 4. III. *Differentiation*. IV. *Materials:* A. For each group of students: 1. **Day 1:** Note-taking Guide and Worksheet. 2. **Day 2:** 3. **Day 3:** 4. **Day 4:** 5. **Day 5:** B. Additional materials: 1. None.
Instructional Procedures	I. **Day 1:**

	A. *Warm-up* (**10 minutes**). The shape of the window shown below is made up of a square and a semicircle.  What is the area of the window to the nearest square inch? Show your work or explain how you know. $$508.68\ in.^2 + 1296\ in.^2 = 1804.68\ in.^2 \cong 1805\ in.^2$$ B. *Launch* (**15 minutes**). I will pass out the note-taking guide and worksheet and cover the definitions I want students to add to their glossaries. 1. **Proof:** A convincing argument that uses logic to show that a statement is true. **2. Euler Diagram:** a. Draw an Euler Diagram for the statement, "If Dave is an NFL football player, then he drives a Porsche."



b. Is this a true statement? Why or why not? *Not true because not all Porsche drives are NFL players.*

Conditional Statement: A statement that can be written in the form "If *p* then *q*," where *p* is the hypothesis and *q* is the conclusion.

Converse of a Conditional Statement: The statement formed by interchanging the hypothesis and the conclusion of a conditional.

Bi-conditional Statement: A statement which can be expressed both as a conditional and a converse; both of which are true. Normally written as *"p* if and only if *q".*

C. *Engagement* (**40 minutes**).

1. First Activity (**10 minutes**): Working in groups, students will complete the reverse side of the notetaking guide.

2. I will then introduce the concept of the **logical chain**. A logical chain is composed of different conditionals linked together to lead one to a conclusion. We use logical chains to construct proofs.

"Consider the following (on note-taking guide).*"*

a. If a number is divisible by 4, then it is divisible by 2.
b. If a number is divisible by 2, then the number is even.
c. If a number is even, then the last digit is 0, 2, 4, 6, or 8.

Conclusion: If a number is divisible by 4, then the last digit is 0, 2, 4,

6, or 8.

"We have used this logical chain to prove the conclusion."

3. Second Activity (**10 minutes**): Students, working in groups, will complete the algebraic inequality exercise (number 2) on the work sheet. We will review when complete.

4. Third Activity (**20 minutes**): Students, working in groups, will complete the worksheet – in class or for homework.

D. *Closing* (**5 minutes**). I will solicit the following input, *"Did you understand the concepts discussed today? Was the pace of the class too fast, too slow, just right?"*

II. **Day 2: Objective** – Link the concepts of conditional statements and logical chains to proofs. Develop understanding of the use of two-column proofs to prove geometric concepts.

A. *Warm-up* (**10 minutes**). Simplify the following expressions:

1. $(3.7x + 2) - (1.7x + 3)$
2. $(-5x + 2y) - (3x + 2y)$
3. $(9v - 8w) - (8v - 9w)$
4. $(2q + 3y) - (-4q + 5) + (6q - 7)$
5. $(9 + 4y) - (-1 + 8y) + (7 - y)$

A. *Homework Review* (**15 minutes**). We will review logical chains, spending some time on it if necessary. I will ask students to come to the board to demonstrate how to do them:

If the police catch Tim speeding, then Tim gets a ticket.
If Tim drives a car, then Tim drives too fast.
If Tim drives too fast, then the police catch Tim speeding.

1. Read all of the statements.

2. Look for the statement that appears to be the logical first one.

If Tim drives a car, then Tim drives too fast.

3. Identify the conclusion of this statement.

Tim drives too fast.

4. Look for the statement that has that conclusion as its hypothesis.

If Tim drives too fast, then the police catch Tim speeding.

5. That is the next statement in the chain.

How to do the final problem.

1. $AB = CD$.

2. Use the Algebraic Properties of Equality to prove that AC = BD.

   Try to set up a logical chain.

   a. If AB = CD, then AB + BC = BC + CD by the Addition Property of Equality.
   b. If AB + BC = BC + CD
   c. ...and if AB + BC = AC and BC + CD = BC
   d. ...then AC = BD.

B. *Launch* (**10 minutes**). "Now we will consider how to do proofs using the two – column format." I will handout the note-taking guide and ask students to complete with me the first two-column proof.

C. *Engagement* ().

1. First Activity (**10 minutes**). Working in pairs, students will complete the second part of the Overlapping Segments Theorem Proof.

2. I will then discuss how we have proved the Overlapping Segments Theorem and lead the class in a discussion of the Overlapping Angles Theorem (**10 minutes**).

3. Second Activity (**10 minutes**). Working in groups, students will

attempt to write the proof for the first part of the Overlapping Angles Theorem.

4. Third Activity (**10 minutes**). Working in groups, students will attempt to write the proof for the second part of the Overlapping Angles Theorem.

D. *Closing* (**5 minutes**). I will then discuss the Equivalence Properties of Equality from the Note-taking Guide and assign the homework.

III. **Day 3:**

A. **Objective:** Continue to build proficiency in performing proofs.

B. *Warm-up* (**10 minutes**). Simplify the following expressions:

1. $(5x + 7) - (2x - 7)$
2. $(5x - 6y) - 2(x + y)$
3. $(5x + 3y - 7) - 3(2x - y)$
4. $8x^2 - (2 - 5x^2)$
5. $\frac{2x+8}{2}$
6. $\frac{5y^2+20}{5}$

C. *Homework Review* (**20 minutes**). See attached answer key. We will take this slowly. Take time to review the Equivalence Property of Equality.

D. *Launch.* We will continue our study of proofs.

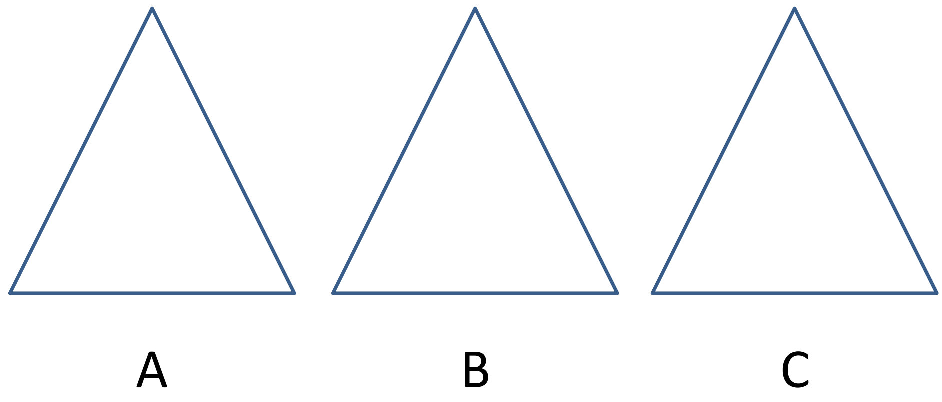
E. *Engagement* ().

1. First Activity (**10 minutes**): Working in groups, students will complete exercises 9 – 12, page 112 of Holt, *Geometry*.

2. Second Activity (**10 minutes**): Working in groups, students will complete exercises 21 – 28, page 113 of Holt, *Geometry*.

3. Third Activity (**15 minutes**): I will introduce the Equivalence Property of Congruence:

   a. For any three figures, *A*, *B*, and *C*:



b. Reflexive Property: $figure\ A \cong figure\ A$.
c. Symmetric Property: $If\ figure\ A \cong figure\ B, then\ figure\ B \cong figure\ A$.
d. Transitive Property: $If\ figure\ A \cong figure\ B, and\ figure\ B \cong figure\ C, then\ figure\ A \cong figure\ C$.

With this property and the others we have learned, students will complete problems 38 – 46 on page 114. I will model 30 – 33:

30. Given.
31. Symmetric Property: If a = b, then b = a.
32. Substitution Property.
33. $m\angle BAC = m\angle DEC$.

F. *Closing* (**5 minutes**). I will state, *"Complete the problems assigned in class and try 38 – 46 for homework tonight."*

IV. **Day 4:**

A. *Warm-up* (**10 minutes**).

1. Write the following statement as a conditional statement.
   Write the converse of the conditional statement.
   Is it bi conditional?

   Angles whose measures add to 180° are supplementary angles.

2. Draw and Euler Diagram to illustrate the conditional statement.

B. *Homework Review* (**20 minutes**).

1. Addition Property.
   Division Property.

2. Transitive Property.

3. Symmetric Property.
   Addition Property.

4. $m\angle EDG = 9x - 8$
   $m\angle FDG = 6x + 8$
   $m\angle GDH = m\angle EDF = 2x - 4$
   $(2x - 4) + (6x + 8) = (9x - 8)$
   $x = 12.$
   $m\angle HDF = (2x - 4) + (6x + 8) = 20 + 80 = 100°$
   $m\angle HDG = 20°$

C. *Launch.* We will continue with our study of proofs.

D. *Engagement.*

1. First Activity (**10 minutes**): Quiz review using attached review sheet / notetaking guide.

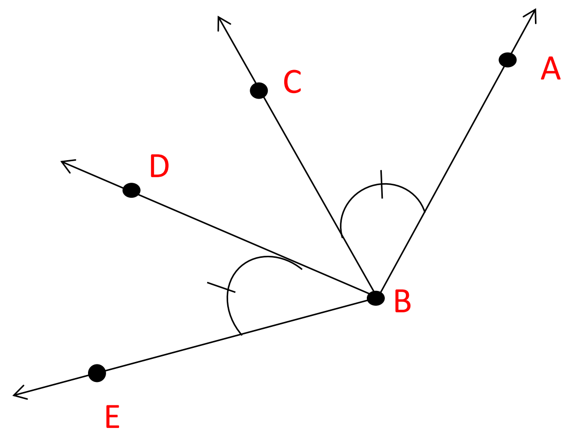
2. Second Activity (**20 minutes**): The previous activity will set the stage for this one. With the properties we have learned, students will complete problems 38 – 46 on page 114. I will model 30 – 33:

   30. Given.
   31. Symmetric Property: If a = b, then b = a.
   32. Substitution Property.
   33. $m\angle BAC = m\angle DEC$.

E. *Closing* (**5 minutes**): I will remind students of tomorrow's quiz and tell them I will be available at homework help.

V. **Day 5:**

A. *Warm-up* (**10 minutes**). Consider the following angles:



$m\angle ABC = m\angle DBE$
$m\angle ABC = 3x - 1$
$m\angle CBD = 2x + 2$
$m\angle ABD = 6x - 4$

Solve for x:

What is the measure of ∠DBE?

B. *Homework Review* (**15 minutes**).

30. Given.
31. Symmetric Property: If a = b, then b = a.
32. Substitution Property.
33. $m\angle BAC = m\angle DEC$.

38. Angle Addition Postulate.
39. 90°.
40. Given.
41. $m\angle CDE + m\angle CDB = m\angle EDB$
42. Transitive or Substitution Property.
43. Transitive or Substitution Property.
44. Subtraction Property.
45. Substitution Property.
46. $m\angle ABC = m\angle EDC$

	C. *Quiz* (**40 minutes**). D. *Closing* (**5 minutes**). Have a good weekend.
Assessment	1. Worksheets. 2. Quiz.

Name: ________________________

Class:_________________________

Date:_________________________

Notetaking Guide – Deductive Reasoning and Proofs – Page 1

I. **Definitions:**

A. **Proof:** A ____________________argument that uses _______________ to show that a statement is ________________________.

B. **Euler Diagram:**

Draw an Euler Diagram for the statement, "If Dave is an NFL football player, then he drives a Porsche."

Is this a true statement? Why or why not?

C. **Conditional Statement:** A statement that can be written in the form _________________ where p is the ________________________ and q is the ____________________________.

D. **Converse of a Conditional Statement:** The statement formed by interchanging the ____ of a conditional.

E. **Definition of a Bi-conditional Statement:** A statement which can be expressed both as a ________________________________; both of which are true. Normally written __________________________________.

Notetaking Guide – Deductive Reasoning and Proofs – Page 2

II. **Examples / Exercises:** For each of the following statements:

- Draw and Euler Diagram illustrating the statement.
- Re-write it in the conditional form.
- Write its converse.
- State whether it is bi-conditional.

A. A lineman is a football player.

1. Euler Diagram:

2. Conditional:

3. Converse:

4. Is this bi-conditional?

B. A hybrid car is a car that is powered by both electricity and gasoline.

1. Euler Diagram:

2. Conditional:

3. Converse:

4. Is this bi-conditional?

Name: ____

Proving Arithmetic Properties

1. Prove the Commutative Property of Addition.

   Prove: $a + b = b + a$

   Given: $a + b = c$
   $b + a = c$

2. Prove the Commutative Property of Multiplication.

   Prove: $a * b = b * a$

   Given: $a * b = c$
   $b * a = d$
   $c = d$

3. Prove the Associative Property of Addition.

   Prove: $a + (b + c) = (a + b) + c$

   Given: $a + (b + c) = e$
   $(a + b) + c = d$
   $e = d$

4. Prove the Additive Inverse Property.

Prove: $a + (-a) = 0$

Given: $a + 0 = a$

Name: ___________________________

Class:____________________________

Date:____________________________

Logical Chains and Proofs

1. Consider the following logical chain:

   a. If a number is divisible by 4, then the number is divisible by 2.
   b. If a number is divisible by 2, then the number is even.
   c. If a number is even, then the last digit is 0, 2, 4, 6, or 8.

   Conclusion: If a number is divisible by 4, then the last digit is 0, 2, 4, 6, or 8.

2. Arrange each set of statements to form a logical chain. Then write the conditional statement that follows from the logical chain.

   a. If it is cold, then birds fly south.
   If the days are short, then it is cold.
   If it is winter, then the days are short.

   b. If the police catch Tim speeding, then Tim gets a ticket.
   If Tim drives a car, then Tim drives too fast.
   If Tim drives too fast, then the police catch Tim speeding.

c. If quompies plaun, then romples gleet.
   If ruskers bleer, then homblers frain.
   If homblers frain, then quompies plaum.

3. Given the following number line:

a. $AB = CD$.

b. Use the Algebraic Properties of Equality to prove that $AC = BD$.

We could prove this using a two – column format:

Statements	**Reasons**
1. $AB = CD$	Given.
2.	
3.	
4.	
5.	

We have proven the statement, *"If AB = CD, then AC = BD."* We could also prove the statement *"If AC = BD, then AB = CD"* in the same way:

Statements	**Reasons**
1. $AC = BD$	Given.
2.	
3.	
4.	
5.	

4. We have just proven the following theorem using the two – column proof format:

Overlapping Segments Theorem

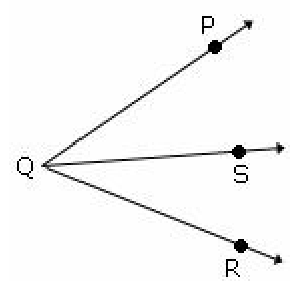
Given a segment with points *A*, *B*, *C*, and *D* arranged as shown, the following statements are true:

a. If $AB = CD$, then $AC = BD$.
b. If $AC = BD$, then $AB = CD$.

5. Now let's consider the following postulate:

Angle Addition Postulate

If a point S lies in the interior of ∠PQR, then ∠PQS + ∠SQR = ∠PQR.



More about Angle Addition Postulate

If the sum of the two angles measure up to 90°, then the angles are called to be 'complementary angles'.

If the sum of the two angles measure up to 180°, then the angles are called to be 'supplementary angles'.

The angles sharing a common side are called as 'adjacent angles'.

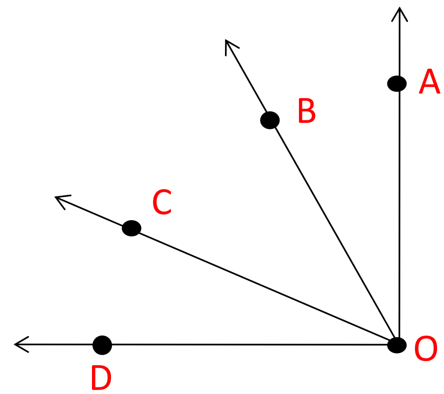
6. Now let's consider another theorem:

Overlapping Angles Theorem

Given ∠AOD with point B and C in its interior as shown, the following statements are true:

a. If m∠AOB = m∠COD, then ____________________.

b. If m∠AOC = m∠BOD, then_____________________.



Prove using the two column format (***Hint***: Recall the angle addition postulate).

Statements	**Reasons**
1. m∠AOB = m∠COD	Given.
2.	
3.	
4.	
5.	

Statements	**Reasons**
1. m∠AOC = m∠BOD	Given.
2.	
3.	
4.	
5.	

Equivalence Properties

1. Complete the following:

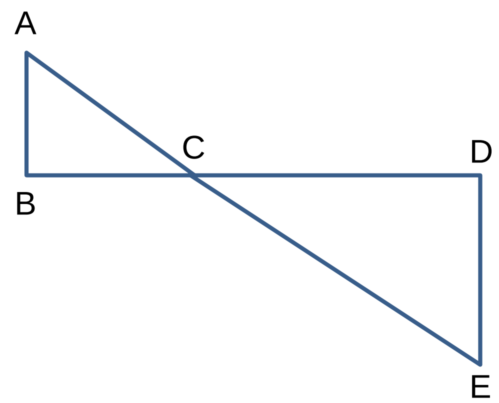
Equivalence Properties of Equality

Reflexive Property	
Symmetric Property	
Transitive Property	

Equivalence Properties of Congruence

Reflexive Property	
Symmetric Property	
Transitive Property	

2. Complete the following proof:



Given: $m\angle BAC + m\angle ACB = 90°$
$m\angle DCE + m\angle DEC = 90°$
$m\angle ACB = m\angle DCE$

Prove: $m\angle BAC = m\angle DEC$

Statements	**Reasons**
1.	
2.	
3.	
4.	
5.	
6.	

Equivalence Properties

1. Consider the following:

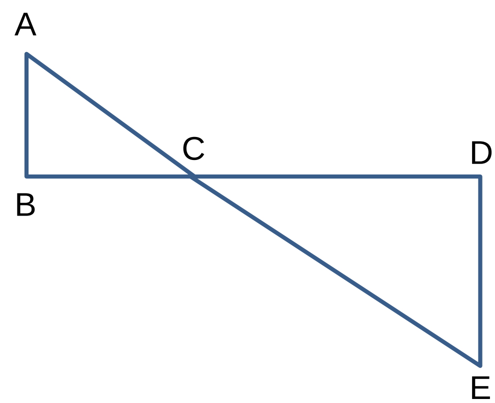
Equivalence Properties of Equality

Reflexive Property	For any real number a, $a = a$
Symmetric Property	For any real numbers a and b, if $a = b$, then $b = a$.
Transitive Property	For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

Equivalence Properties of Congruence

Reflexive Property	$Figure\ A \cong Figure\ A$
Symmetric Property	$If\ Figure\ A \cong Figure\ B\ then\ Figure\ B \cong Figure\ A$
Transitive Property	$If\ Figure\ A \cong Figure\ B\ and\ Figure\ B \cong Figure\ C\ then\ Figure\ A \cong Figure\ C$

2. Complete the following proof:



Given: $m\angle BAC + m\angle ACB = 90°$
$m\angle DCE + m\angle DEC = 90°$
$m\angle ACB = m\angle DCE$

Prove: $m\angle BAC = m\angle DEC$

Statements	**Reasons**
7. $m\angle BAC + m\angle ACB = 90°$	Given.
8. $m\angle DCE + m\angle DEC = 90°$	Given.
9. $m\angle BAC + m\angle ACB = m\angle DCE + m\angle DEC$	Transitive Property of Equivalence.
10. $m\angle ACB = m\angle DCE$	Given.
11. $m\angle BAC + m\angle ACB = m\angle ACB + m\angle DEC$	Substitution Property.
12. $m\angle BAC = m\angle DEC$	Subtraction Property.

Name: ___________________________

Class:____________________________

Date:____________________________

Notetaking Guide Key – Page 1

a. Given a segment with points *A*, *B*, *C*, and *D*, and
b. Given that $AB = CD$,
c. Prove that $AC = BD$.

1. We could prove this using a two – column format:

Statements	**Reasons**
1. $AB = CD$	Given.
2. $AB + BC = BC + CD$	Addition Property of Equality
3. $AB + BC = AC$	Segment Addition Postulate
4. $BC + CD = BD$	Segment Addition Postulate
5. $AC = BD$	Substitution Property of Equality

We have proven the statement, *"If AB = CD, then AC = BD."* We could also prove the statement *"If AC = BD, then AB = CD"* in the same way:

Statements	**Reasons**
1. $AC = BD$	Given.
2. $BC + CD = BD$	Segment Addition Postulate
3. $AB + BC = AC$	Segment Addition Postulate
4. $AB + BC = BC + CD$	Substitution Property of Equality
5. $AB = CD$	Addition Property of Equality

2. We have just proven the following theorem using the two – column proof format:

Overlapping Segments Theorem

Given a segment with points *A*, *B*, *C*, and *D* arranged as shown, the following statements are true:

a. If *AB* = *CD*, then *AC* = *BD*.
b. If *AC* = *BD*, then *AB* = *CD*.

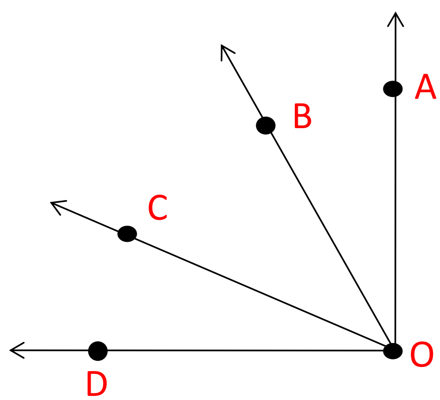
3. Now let's consider another theorem:

Overlapping Angles Theorem

Given ∠AOD with point B and C in its interior as shown, the following statements are true:

a. If m∠AOB = m∠COD, then m∠AOC = m∠BOD.

b. If m∠AOC = m∠BOD, then m∠AOB = m∠COD.



Prove using the two column format (***Hint***: Recall the angle addition postulate).

Statements	**Reasons**
1. m∠AOB = m∠COD	Given.
2. m∠AOB + m∠BOC = m∠COD + m∠BOC	Addition Property of Equality
3. m∠AOB + m∠BOC = m∠AOC	Angle Addition Postulate
4. m∠COD + m∠BOC = m∠BOD	Angle Addition Postulate
5. m∠AOC = m∠BOD	Transitive Property of Equality.

Statements	**Reasons**
1. m∠AOC = m∠BOD	Given.
2. m∠COD + m∠BOC = m∠BOD	Angle Addition Postulate
3. m∠AOB + m∠BOC = m∠AOC	Angle Addition Postulate
4. m∠AOB + m∠BOC = m∠COD + m∠BOC	Addition Property of Equality
5. m∠AOB = m∠COD	Transitive Property of Equality.

3. Consider the following:

Equivalence Properties of Equality

Reflexive Property	For any real number a, $a = a$
Symmetric Property	For any real numbers a and b, if $a = b$, then $b = a$.
Transitive Property	For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

Write an example illustrating each property.

1. Reflexive: 4 = 4.
2. Symmetric: $9 = 3^2, 3^2 = 9$
3. Transitive: $9 = 3^2, 3^2 = 6 + 3, 9 = 6 + 3$

Review Sheet - Introduction to Proofs

1. Definitions: Review the definitions we completed on the Notetaking Sheet on Monday.

2. Properties and Theorems:

Algebraic Properties of Equality

Addition Property	
Subtraction Property	
Multiplication Property	
Division Property	
Substitution Property	

Equivalence Properties of Equality

Reflexive Property	
Symmetric Property	
Transitive Property	

Overlapping Segments Theorem

Given a segment with points A, B, C, and D arranged as shown, the following statements are true:

a. If $AB = CD$, then ____________.

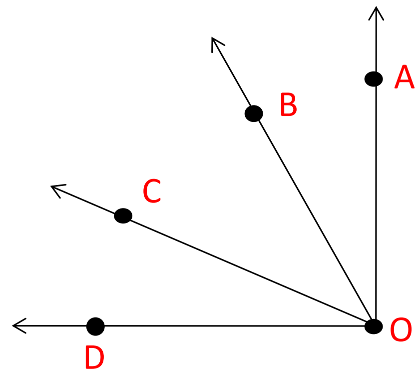
b. If $AC = BD$, then ____________.

Overlapping Angles Theorem

Given ∠AOD with point B and C in its interior as shown, the following statements are true:

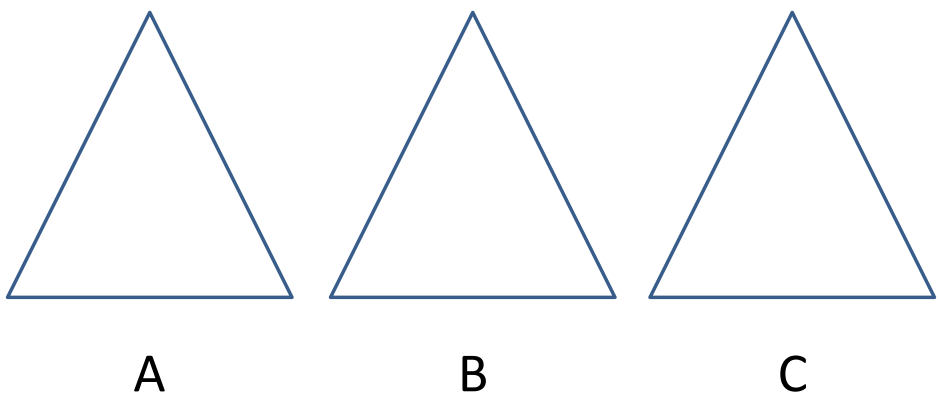
a. If m∠AOB = m∠COD, then ________________.

b. If m∠AOC = m∠BOD, then ________________.



Equivalence Properties of Congruence

For any three figures, *A*, *B*, and *C*:



Reflexive Property	
Symmetric Property	
Transitive Property	

3. Review the logical chains exercises from Tuesday.

4. Review the proof for the Overlapping Segments Theorem and the Overlapping Angles Theorem.

Review Sheet – Introduction to Proofs

1. Definitions: Review the definitions we completed on the Notetaking Sheet on Monday.

2. Properties and Theorems:

Algebraic Properties of Equality

Addition Property	$if\ a = b, then\ a + c = b + c$
Subtraction Property	$if\ a = b, then\ a - c = b - c$
Multiplication Property	$if\ a = b, then\ ac = bc$
Division Property	$if\ a = b\ and\ c \neq 0, then \frac{a}{c} = \frac{b}{c}$
Substitution Property	If $a = b$, then if you replace a with b in any true equation containing a then the resulting equation will still be true.

Equivalence Properties of Equality

Reflexive Property	For any real number a, $a = a$
Symmetric Property	For any real numbers a and b, if $a = b$, then $b = a$.
Transitive Property	For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

Overlapping Segments Theorem

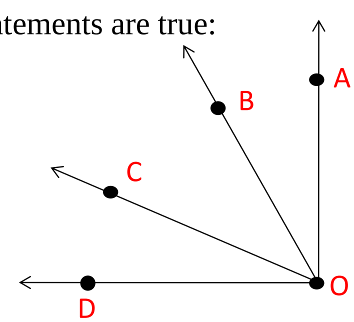
Given a segment with points *A*, *B*, *C*, and *D* arranged as shown, the following statements are true:

a. If *AB* = *CD*, then *AC* = *BD*.
b. If *AC* = *BD*, then *AB* = *CD*.

Overlapping Angles Theorem

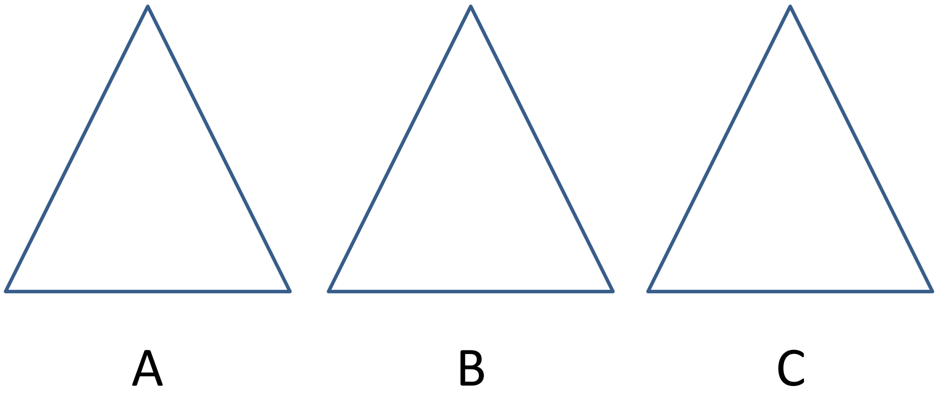
Given ∠AOD with point B and C in its interior as shown, the following statements are true:

1. If m∠AOB = m∠COD, then m∠AOC = m∠BOD.
2. If m∠AOC = m∠BOD, then m∠AOB = m∠COD.



Equivalence Properties of Congruence

For any three figures, *A*, *B*, and *C*:



Reflexive Property	$figure\ A \cong figure\ A$
Symmetric Property	$If\ figure\ A \cong figure\ B, then\ figure\ B \cong figure\ A.$
Transitive Property	$If\ figure\ A \cong figure\ B, and\ figure\ B \cong figure\ C, then\ figure\ A \cong figure\ C.$

3. Review the logical chains exercises from Tuesday.

4. Review the proof for the Overlapping Segments Theorem and the Overlapping Angles Theorem.

Name: ______________________

Class: ______________________

Date: ______________________

Geometry – Quiz 3

1. **Definitions.** Write the definitions of the following algebra terms. (**3 points each**)

   a. **Proof.** ____

   ____

   b. **Conditional Statement.** ____

   ____

   ____

   c. **Converse of a Conditional Statement.** ____

   ____

   ____

   d. **Bi-conditional Statement.** ____

   ____

   ____

2. State the **Equivalence Properties of Equality.** (**3 points each**)

Reflexive Property	
Symmetric Property	
Transitive Property	

3. For this statement, *All sailors visit Hong Kong, China:*

   a. Draw an Euler diagram illustrating this statement. (**4 points**)

   b. Put this in the form of a conditional statement. Underline and label the *hypothesis (p)* and the *conclusion (q)*. (**4 points**)

   c. State the converse. (**4 points**)

   d. Is this bi-conditional? (**3 points**)

4. Arrange the following statement into a logical chain. (**4 points**)

   If the days are longer, then kids can play more. ____________________

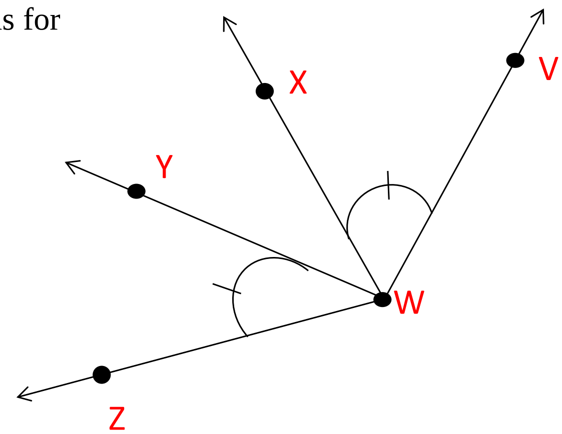
   If kids can play more, then they are happy. ____________________

   If it is summer, then the days are longer. ____________________

5. For the following two column proof below state the reasons for each statement. (**2 points each**)

- Given: m∠VWX = m∠YWZ

- Proved: m∠VWY = m∠XWZ



Statements	**Reasons**
6. m∠VWX = m∠YWZ	a.
7. m∠VWX + m∠XWY = m∠YWZ + m∠XWY	b.
8. m∠VWX + m∠XWY = m∠VWY	c.
9. m∠YWZ + m∠XWY = m∠XWZ	d.
10. m∠VWY = m∠XWZ	e.

Name: ___________________________

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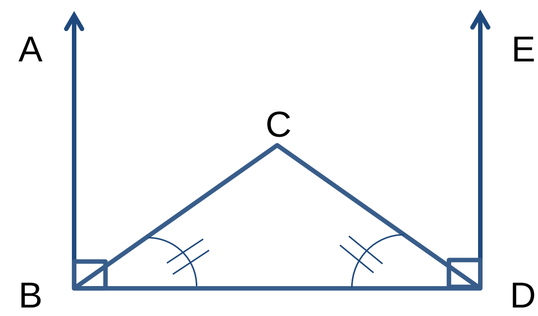
Proof Exercise

Given: $m\angle CBD = m\angle CDB$

$m\angle ABD = 90°$

$m\angle EDB = 90°$

Prove: $m\angle ABC = m\angle EDC$



Statements	**Reasons**
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.
10.	10.

Name: ___________________________

Class:___________________________

Date:___________________________

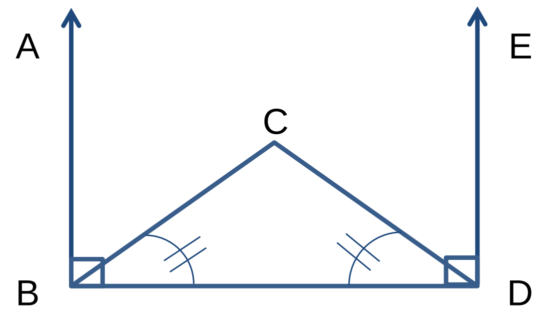
Proof Exercise

Given: $m\angle CBD = m\angle CDB$

$m\angle ABD = 90°$

$m\angle EDB = 90°$

Prove: $m\angle ABC = m\angle EDC$



Statements	**Reasons**
1. $m\angle ABD = 90°$	1. Given
2. $m\angle ABC + m\angle CBD = m\angle ABD$	2. Angle Addition Postulate
3. $m\angle ABC + m\angle CBD = 90°$	3. Transitive Property or Substitution Property
4. $m\angle EDB = 90°$	4. Given
5. $m\angle EDC + m\angle CDB = m\angle EDB$	5. Angle Addition Postulate
6. $m\angle EDC + m\angle CDB = 90°$	6. Transitive Property or Substitution Property
7. $m\angle ABC + m\angle CBD = m\angle EDC + m\angle CDB$	7. Transitive Property or Axiom 6
8. $m\angle CBD = m\angle CDB$	8. Subtraction Property
9. $m\angle ABC + m\angle CBD = m\angle EDC + m\angle CBD$	9. Substitution Property
10. $m\angle ABC = m\angle EDC$	10. Subtraction Property

Name: ___________________________

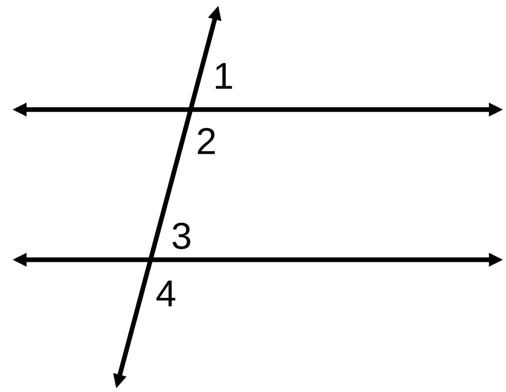
Class:____________________________

Date:____________________________

Proof Exercise

Given: $m\angle 1 = m\angle 3$

Prove: $m\angle 2 = m\angle 4$



Statements	**Reasons**

Name: ___________________________

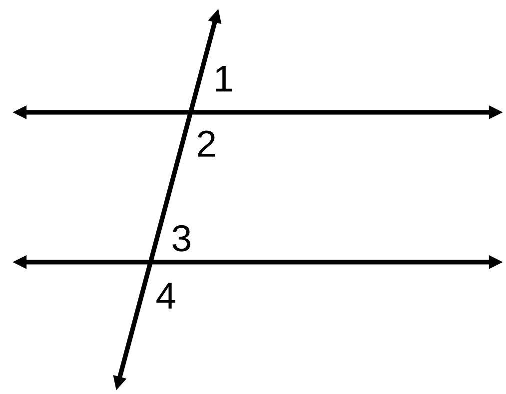
Class:___________________________

Date:___________________________

Proof Exercise

Given: $m\angle 1 = m\angle 3$

Prove: $m\angle 2 = m\angle 4$



Statements	**Reasons**
$m\angle 1 = m\angle 3$	Given
$m\angle 1 + m\angle 2 = 180°$	Linear Pair Property
$m\angle 3 + m\angle 4 = 180°$	Linear Pair Property
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Substitution Property
$m\angle 1 + m\angle 2 = m\angle 1 + m\angle 4$	Substitution Property
$m\angle 2 = m\angle 4$	Subtraction Property

Name: ___________________________

Class:____________________________

Date:____________________________

Proof Exercise

Given: $3(x-4)=48$

Prove: $x=20$

Statements	**Reasons**

Given: $2(x+1)=3x-3$

Prove: $x=5$

Statements	**Reasons**

Given: $x+\frac{5}{9}+\frac{3x}{18}=\frac{2}{6}+5x$

Prove: $x=\frac{4}{69}$

Statements	**Reasons**

Name: ______________________

Class: ______________________

Date: ______________________

Practice Assessment – Logic and Proof

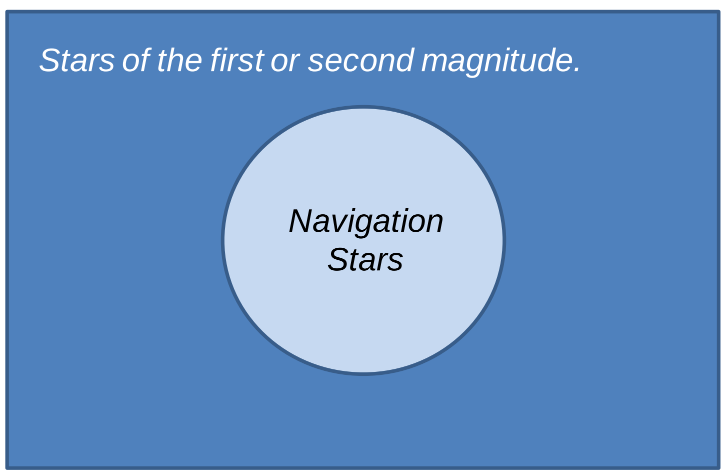
I. **Logic.**

In the following statement:

A. Rewrite the statement as a conditional statement.
B. Underline the hypothesis and the conclusion of the conditional statement.
C. Draw an Euler diagram that illustrates the conditional statement.
D. Write the converse of the conditional statement and state whether it is true.

Navigation Stars are of the first or second magnitude.

- If a star is a Navigation Star, then it is of the first or second magnitude.



- If a star is of the first or second magnitude, then it is a Navigation Star. *This statement is not true.*

II. **Proof.**

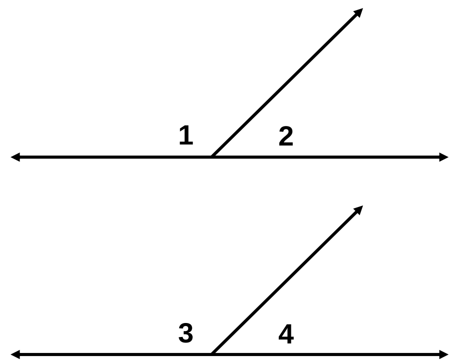
Given: $\angle 1 \cong \angle 3$

$(m\angle 1 = m\angle 3)$

$\angle 1$ and $\angle 2$ are supplementary

$\angle 3$ and $\angle 4$ are supplementary

Prove: $\angle 2 \cong \angle 4$



Statements	**Reasons**
$m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 4 = 180°$	Definition of supplementary angles.
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Transitive Property, Substitution Property, Axiom 6
$m\angle 1 = m\angle 3$	Definition of congruent angles.
$m\angle 1 + m\angle 2 = m\angle 1 + m\angle 4$	Substitution Property
$m\angle 2 = m\angle 4$	Subtraction Property
$\angle 2 \cong \angle 4$	Definition of congruent angles.

Name: ___________________________

Class:___________________________

Date:___________________________

Practice Assessment – Logic and Proof

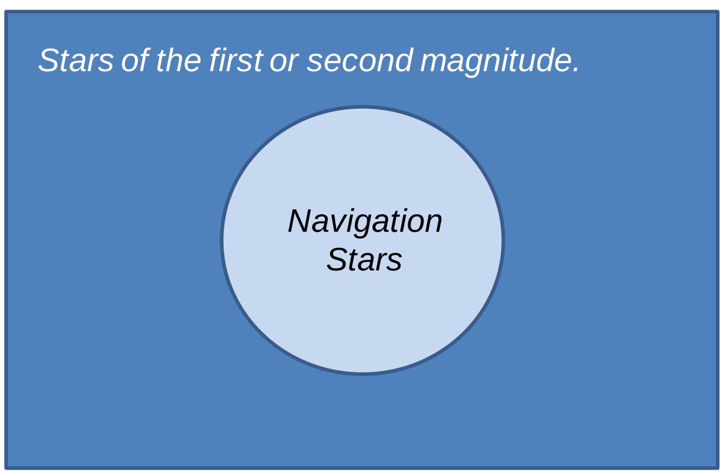
III. Logic.

In the following statement:

E. Rewrite the statement as a conditional statement.
F. Underline the hypothesis and the conclusion of the conditional statement.
G. Draw an Euler diagram that illustrates the conditional statement.
H. Write the converse of the conditional statement and state whether it is true.

Navigation Stars are of the first or second magnitude.

- If a star is a Navigation Star, then it is of the first or second magnitude.



- If a star is of the first or second magnitude, then it is a Navigation Star. *This statement is not true.*

IV. **Proof.**

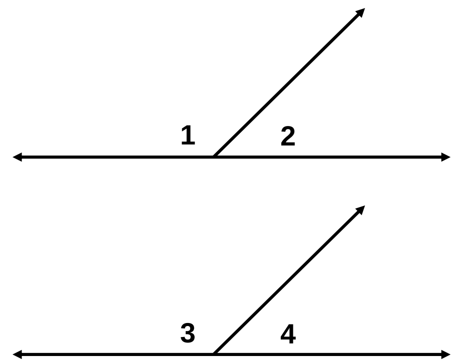
Given: $\angle 1 \cong \angle 3$

$(m\angle 1 = m\angle 3)$

$\angle 1$ and $\angle 2$ are supplementary

$\angle 3$ and $\angle 4$ are supplementary

Prove: $\angle 2 \cong \angle 4$



Statements	**Reasons**
$m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 4 = 180°$	Definition of supplementary angles.
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Transitive Property, Substitution Property, Axiom 6
$m\angle 1 = m\angle 3$	Definition of congruent angles.
$m\angle 1 + m\angle 2 = m\angle 1 + m\angle 4$	Substitution Property
$m\angle 2 = m\angle 4$	Subtraction Property
$\angle 2 \cong \angle 4$	Definition of congruent angles.

Name: ___________________________

Class:____________________________

Date:____________________________

Practice Assessment – Logic and Proof

I. Logic.

In the following statement:

A. Rewrite the statement as a conditional statement.
B. Underline the hypothesis and the conclusion of the conditional statement.
C. Draw an Euler diagram that illustrates the conditional statement.
D. Write the converse of the conditional statement and state whether it is true.

Navigation Stars are of the first or second magnitude.

II. **Proof.**

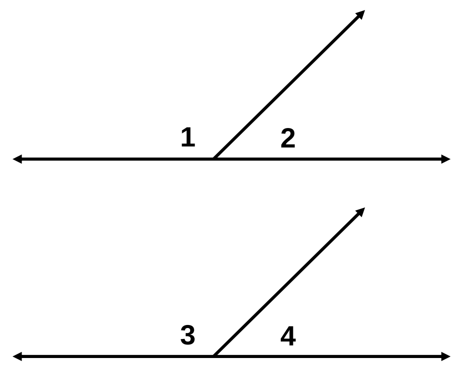
Given: $\angle 1 \cong \angle 3$

$(m\angle 1 = m\angle 3)$

$\angle 1$ and $\angle 2$ are supplementary

$\angle 3$ and $\angle 4$ are supplementary

Prove: $\angle 2 \cong \angle 4$



Statements	Reasons

Name: ________________________

Class:_________________________

Date:_________________________

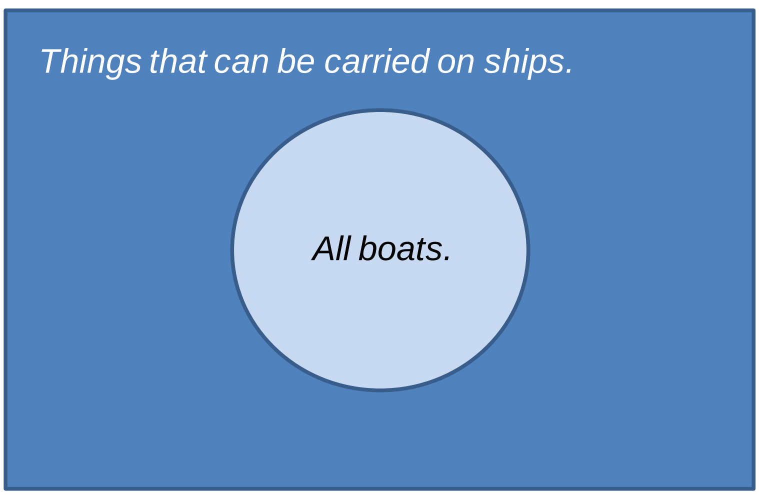
Assessment – Logic and Proof

I. Logic.

In the following statement:

A. Rewrite the statement as a conditional statement.
B. Underline the hypothesis and the conclusion of the conditional statement.
C. Draw an Euler diagram that illustrates the conditional statement.
D. Write the converse of the conditional statement and state whether it is true.

Boats can be carried on ships.



If it is a boat then it can be carried on a ship.

If it is a ship, then it can be carried on a boat (untrue).

II. **Proof.**

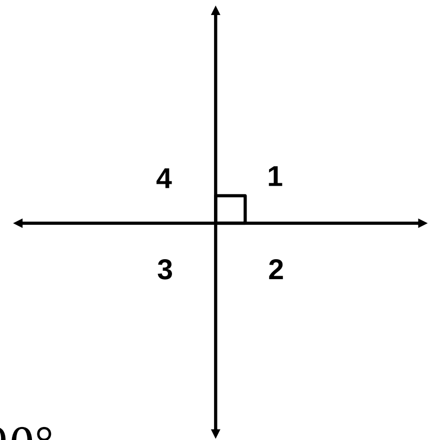
Given: $\angle 1$ is a right angle

$\angle 1$ and $\angle 2$ are supplementary

$\angle 3$ and $\angle 4$ are supplementary

$m\angle 1 = m\angle 3$

Prove: $m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = 90°$



Statements	Reasons
$\angle 1$ is a right angle	Given
$m\angle 1 = 90°$	Definition of a right angle
$m\angle 1 = m\angle 3$	Given
$m\angle 3 = 90°$	Substitution, Transitive, Axiom 6
$m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 4 = 180°$	Definition of supplementary angles.
$90° + m\angle 2 = 180°$ $90° + m\angle 4 = 180°$	Substitution Property
$m\angle 2 = 90°$ $m\angle 4 = 90°$	Subtraction Property
$m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = 90°$	Substitution Property

Name: ___________________________

Class:____________________________

Date:____________________________

Assessment – Logic and Proof

I. Logic.

In the following statement:

A. Rewrite the statement as a conditional statement.
B. Underline the hypothesis and the conclusion of the conditional statement.
C. Draw an Euler diagram that illustrates the conditional statement.
D. Write the converse of the conditional statement and state whether it is true.

Boats can be carried on ships.

II. **Proof.**

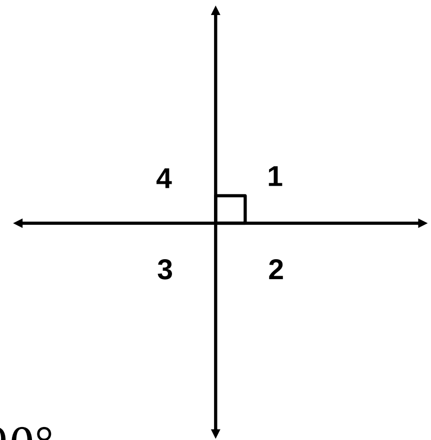
Given: $\angle 1$ is a right angle

$\angle 1$ and $\angle 2$ are supplementary

$\angle 3$ and $\angle 4$ are supplementary

$m\angle 1 = m\angle 3$

Prove: $m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = 90°$



Statements	**Reasons**

Name: ____________________________

Date:____________________________

Advisor: ____________________________

Geometry Assessment – Geometric Proof

Place all answers and show all work on separate sheets of paper.

1. Change the following conditional statements into the "Given" and "Prove" of a two – column proof:

   a. If the world had been flat, then Christopher Columbus would have fallen off.

   b. If $m\angle A = 125°$ and $m\angle B = 55°$ then ∠A and ∠B are supplementary.

   c. If $2 + 2 = 10$, then $2 + 1 = 3$.

   d. If *Rentz* burns 44.5 gallons of fuel per nautical mile and has traveled 1000 nautical miles, then *Rentz* has burned 44,500 gallons of fuel.

2. Change the following "Given" and "Prove" statements to conditional statements:

   a. Given: Man is meant to fly.
   Prove: Man has wings.

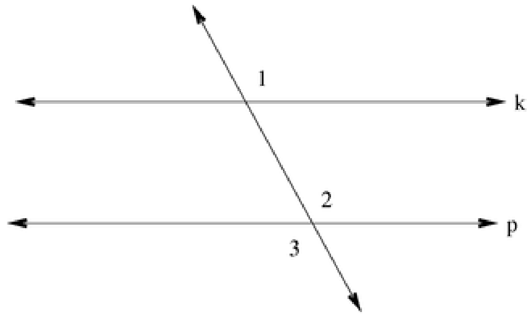
   b. Given: $2x + 17 = 29$
   Prove: $x = 6$

   c. Given: All Caucasians have pale skin tone.
   Brenda has a pale skin tone.
   Prove: Brenda is a Caucasian.

   d. Given: The *Enterprise* is a Federation starship.
   Federation starships are commanded by Starfleet Captains.
   James Kirk commands the *Enterprise*.
   Prove: James Kirk is a Starfleet Captain.

3. Complete the following two – column proofs using the given statements and reasons.

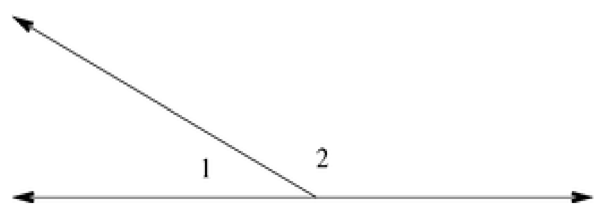
   a. Given: line k || line p
      Prove: $m\angle 1 = m\angle 3$



      Statements: Reasons:

      $m\angle 2 = m\angle 3$ If 2 || lines are cut by a transversal, then corresponding
      $m\angle 1 = m\angle 2$ ∠'s are = in measure.
      $m\angle 1 = m\angle 3$ Vertical ∠'s are = in measure.
      line k || line p Given
      Transitive Property of Equality.

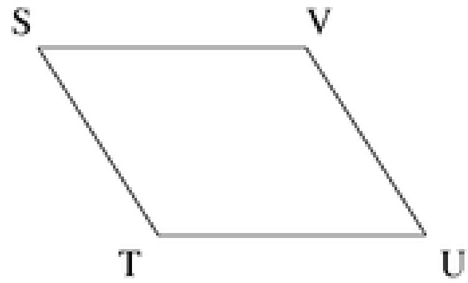
   b. Given: $m\angle 1 = 35°$
      $m\angle 2 = 145°$
      Prove: ∠1 and ∠2 are supplementary.



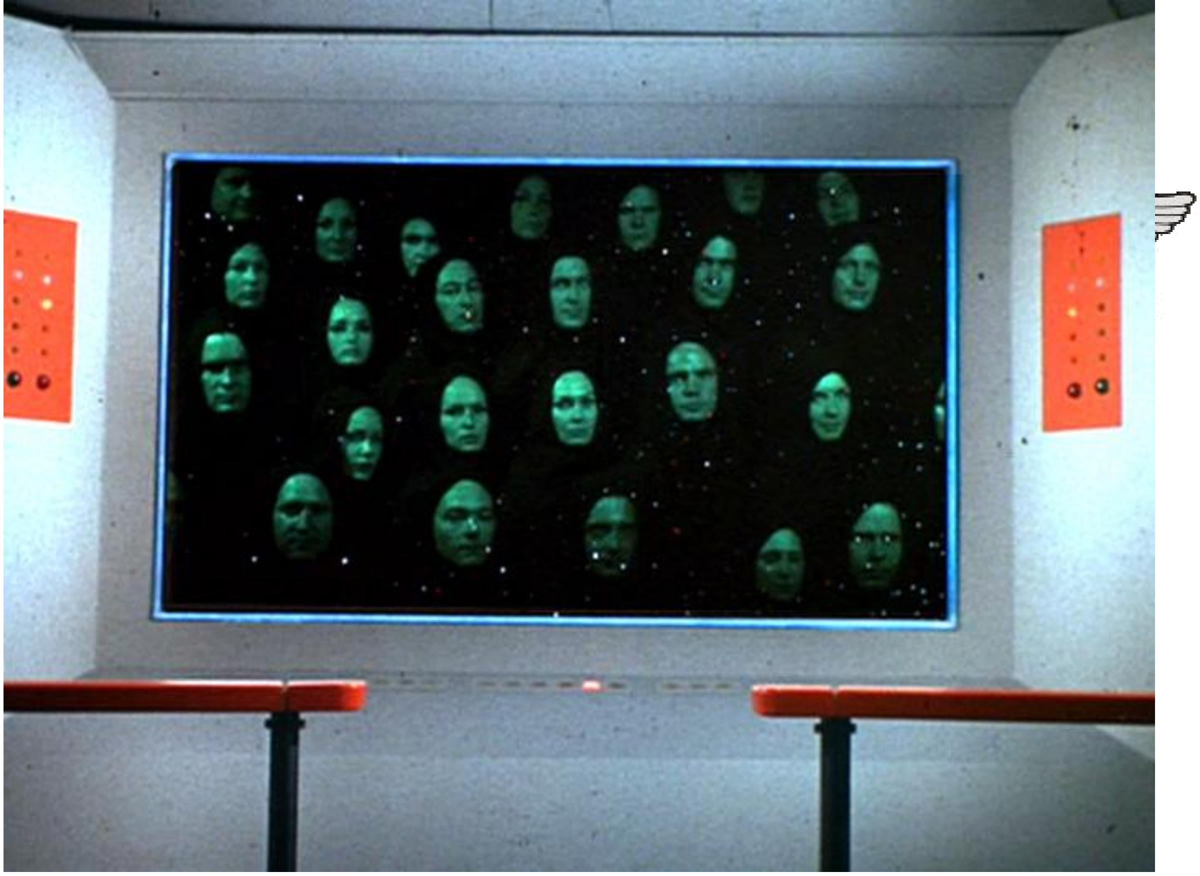
      Statements: Reasons:
      $m\angle 1 + m\angle 2 = 35° + 145°$ Given.
      $m\angle 1 + m\angle 2 = 180°$ Substitution Property of Equality.
      $m\angle 1 = 35°$ Definition of Supplementary Angles.
      $m\angle 2 = 145°$ Addition Property of Equality.
      ∠1 and ∠2 are supplementary. Given.

c. Given: Parallelogram STUV
$m\angle U = 42°$

Prove: $m\angle T = 138°$



Statements:	Reasons:
$m\angle U = 42°$	Given
$m\angle U + m\angle T = 180°$	Substitution
$42° + m\angle T = 180°$	Subtraction
Parallelogram STUV	If a quadrilateral is a Parallelogram, then consecutive ∠'s are supplementary.
$m\angle T = 138°$	Given
∠U and ∠T are supplementary angles.	Definition of supplementary



Exponents and Exponential Functions

9^{th} Grade Mathematics

Captain Thomas R. Beall, U. S. Navy (Ret.)

Slide 1

Review of Exponents

For all real numbers *x* and all positive integers *n*:

$$x^n = \underbrace{x * x * x * x * \ldots * x}_{n \text{ times}}$$

$2^5 = 2 * 2 * 2 * 2 * 2 = 32$

$12^4 = 12 * 12 * 12 * 12 = 20736$

Slide 2

Review of Exponents

For all real numbers *x* and all integers *m* and *n*:

$$x^m * x^n = x^{m+n}$$

$2^3 * 2^4 = 2^{3+4} = 2^7 = 2 * 2 * 2 * 2 * 2 * 2 * 2 = 128$

$y^3 * y^4 = y^{3+4} = y^7$

$5^y * 5^x = 5^{y+x}$

Slide 3

Review of Exponents

Definition of a Monomial

A monomial is an algebraic expression that is either a constant, a variable, or a product of a constant and one or more variables. The constant is called the ***coefficient.***

$(5t)(-30t^2) = (5)(-30)(t)(t^2) = -150t^{1+2} = -150t^3$

$(-4a^2b)(-ac^2)(3b^2c^2) =$

$(-4)(-1)(3)(a^2)(b)(b^2)(c^2)(c^2) = 12a^3b^3c^4$

Slide 4

Review of Exponents

For all nonzero real numbers x and y, and all integers n:

$$(xy)^n = x^n y^n$$

$(2 \times 5)^5 = 2^5 \times 5^5 = 100000$

Even powers of -1 are equal to 1
Odd powers of -1 are equal to -1

$(-t)^5 = (-1 \times t)^5 = (-1)^5 \times t^5 = -t^5$

Slide 5

Review of Exponents

For all nonzero real numbers x and all integers m and n *where* $m > n$:

$$\frac{x^m}{x^n} = x^{m-n}$$

$\frac{2^6}{2^4} = 2^{6-4} = 2^2 = 4$

$\frac{x^{a+b}}{x^c} = x^{a+b-c}$

Slide 6

Review of Exponents

For all real numbers a, b, and n where $n \geq 0$ and $b \neq 0$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$

$\left(\frac{x^n}{y^n}\right)^2 = \frac{(x^n)^2}{(y^n)^2} = \frac{x^{2n}}{y^{2n}}$

Slide 7

Review of Exponents

For all non-zero real numbers x, and all integers n,

$$x^{-n} = \frac{1}{x^n}$$

$2^{-3} \times 2^2 =$

For any non-zero real number x,

$$x^0 = 1$$

$c^{-4} \times c^4 =$

Slide 8

Scientific Notation

Decimal	Expanded Form	Scientific Notation
1200	1.2 x 1000	1.2×10^{3}
120	1.2 x 100	1.2×10^{2}
12	1.2 x 10	1.2×10^{1}
1.2	1.2 x 1	1.2×10^{0}
0.12	1.2 x 1/10	1.2×10^{-1}
0.012	1.2 x 1/100	1.2×10^{-2}
0.0012	1.2 x 1/1000	1.2×10^{-3}

Slide 9

Problem

The star Betelgeuse is 6×10^{15} miles from Earth. If you traveled at a speed of 1000 miles per hour, how long would it take to get from Earth to Betelgeuse?

$$(6 \times 10^{15}) \div 1000 = \frac{6 \times 10^{15}}{10^3} = 6 \times 10^{15-3} = 6 \times 10^{12} \text{ miles}$$

Slide 10

Problem

A typical X-ray has a wavelength of 0.000000000125 meters. What is this number in scientific notation?

Since the first factor must be a number between 1 and 10, place the decimal after the 1. Because the decimal point moves 10 places to the right, use 10^{-10} as the factor to maintain equality with the original number.

$$0.000000000125 = 1.25 \times 10^{-10}$$

Slide 11

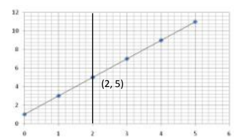
Functions

Definition

Function: A pairing between two sets of numbers in which each element of the first set is paired with no more than one element of the second set.

$2y = 4x + 2$

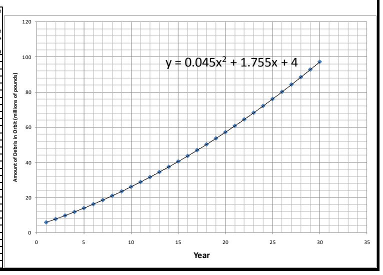
x	y
0	1
1	3
2	5
3	7
4	9
5	11



Slide 12

Exponential Functions

Year	Amount of Debris in Orbit (millions of pounds)
1 (1990)	5.8
2 (1991)	7.69
3 (1992)	9.67
4 (1993)	11.74
5 (1994)	13.9
6 (1995)	16.15
7 (1996)	18.49
8 (1997)	20.92
9 (1998)	23.44
10 (1999)	26.05
11 (2000)	28.75
12 (2001)	31.54
13 (2002)	34.42
14 (2003)	37.39
15 (2004)	40.45
16 (2005)	43.6
17 (2006)	46.84
18 (2007)	50.17
19 (2008)	53.59
20 (2009)	57.1
21 (2010)	60.7
22 (2011)	64.39
23 (2012)	68.17
24 (2013)	72.04
25 (2014)	76
26 (2015)	80.05
27 (2016)	84.19
28 (2017)	88.42
29 (2018)	92.74
30 (2019)	97.15



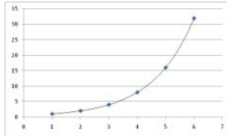
Slide 13

Exponential Functions

Exponential functions are used to model and predict things like population growth and money investment growth.

$y = 2^x$

x	Function	y
0	2^0	1
1	2^1	2
2	2^2	4
3		
4		
5		

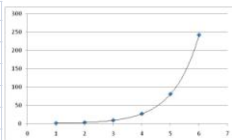


Slide 14

Exponential Functions

$y = 3^x$

x	Function	y
0		
1		
2		
3		
4		
5		



Slide 15

Modeling Growth

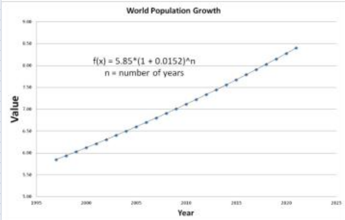
Build a spreadsheet to estimate the increase in the world's population from **1997** to **2011.** Start with the 1997 population of 5.85 billion. Assuming that the population increases by 1.52% each year, the new population each year will be (100 + 1.52%) or (1 + 0.0152) times the previous year's population. So use (1 + 0.0152) as a multiplier.

Year (x)	Process	Exponential Notation (y)	Population in Billions
1997	5.85	5.85*(1 + 0.0152)^0	5.85
1998	5.85 (1 + 0.0152)	5.85*(1 + 0.0152)^1	5.94
1999	[5.85(1 + 0.0152)](1 + 0.0152)	5.85*(1 + 0.0152)^2	6.03
2000	[[5.85(1 + 0.0152)(1 + 0.0152)](1 + 0.0152)	5.85*(1 + 0.0152)^3	6.12
...			

Slide 16

Modeling Growth

Year	Value
1997	5.85
1998	5.94
1999	6.03
2000	6.12
2001	6.21
2002	6.31
2003	6.40
2004	6.50
2005	6.60
2006	6.70
2007	6.80
2008	6.91
2009	7.01
2010	7.12
2011	7.23
2012	7.34
2013	7.45
2014	7.56
2015	7.68
2016	7.79
2017	7.91
2018	8.03
2019	8.15
2020	8.28
2021	8.40



Slide 17

Modeling Growth

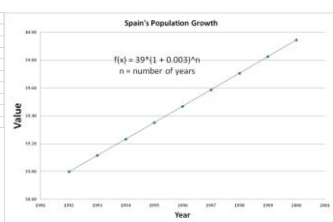
Build a spreadsheet to estimate the increase in Spain's population from **1992** to **2000.** Start with the 1992 population of 39 million. Assuming that the population increases by 0.3% each year, the new population each year will be 100 + 0.3% or 1 + 0.003 times the previous year's population. So use (1 + 0.003) as a multiplier.

Year (x)	Function (y)	Population in Millions
1992		
1993		
1994		
1995		
…		

Slide 18

Modeling Growth

Year	Value
1992	39.00
1993	39.12
1994	39.23
1995	39.35
1996	39.47
1997	39.59
1998	39.71
1999	39.83
2000	39.95



Slide 19

Modeling Growth

Mike sold an antique car for $32,000. The car's value has been growing at a rate of 7% (0.07) per year for the last 25 years. What was the car valued at 25 years ago?

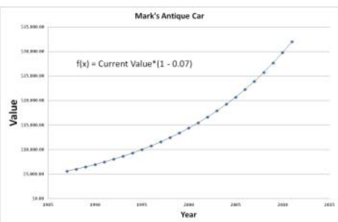
Year (x)	Function (y)	Car Value
2011		
2010		
2009		
2008		
…		

Build a graph that answers this question.

Slide 20

Exponential Functions

Year	Value
2011	$32,000.00
2010	$29,760.00
2009	$27,676.80
2008	$25,739.42
2007	$23,937.66
2006	$22,262.03
2005	$20,703.69
2004	$19,254.43
2003	$17,906.62
2002	$16,653.15
2001	$15,487.43
2000	$14,403.31
1999	$13,395.08
1998	$12,457.43
1997	$11,585.41
1996	$10,774.43
1995	$10,020.22
1994	$9,318.80
1993	$8,666.49
1992	$8,059.83
1991	$7,495.64
1990	$6,970.95
1989	$6,482.98
1988	$6,029.17
1987	$5,607.13

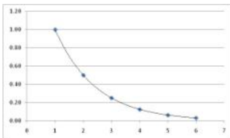


Slide 21

Exponential Functions

$y = 0.5^x$

x	Function	y
0		
1		
2		
3		
4		
5		



Slide 22

Modeling Growth

The population of a city in the United States was 154,494 in 2000 and is decreasing at a rate of 1% (0.01) per year. When will the city's population fall below 140,000?

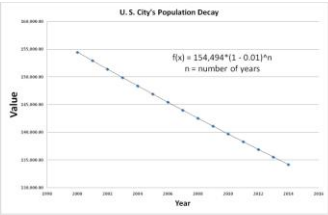
Year (x)	Function (y)	Population in Billions
2000	154,494*(1 – 0.01)^0	?
2001	154,494*(1 – 0.01)^1	?
2001	154,494*(1 – 0.01)^2	?

Build a graph that answers this question.

Slide 23

Modeling Growth

Year	Value
2000	154,494.00
2001	152,949.06
2002	151,419.57
2003	149,905.37
2004	148,406.32
2005	146,922.26
2006	145,453.03
2007	143,998.50
2008	142,558.52
2009	141,132.93
2010	139,721.60
2011	138,324.39
2012	136,941.14
2013	135,571.73
2014	134,216.02



Slide 24

General Growth or Decay Function

General Growth Function:

$$Annual\ Growth\ (G) = Starting\ Number\ (P) * (1 + amount\ of\ change\ each\ year (r))^{number\ \ of\ years\ \ (t)}$$

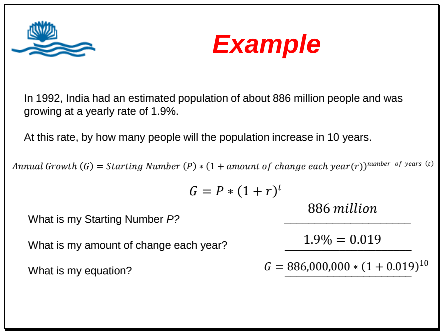
$$G = P * (1 + r)^t$$

General Decay Function:

$$Annual\ Decay\ (D) = Starting\ Number\ (P) * (1 + amount\ of\ change\ each\ year (r))^{number\ \ of\ years\ \ (t)}$$

$$D = P * (1 - r)^t$$

Slide 25


Example
In 1992, India had an estimated population of about 886 million people and was growing at a yearly rate of 1.9%.
At this rate, by how many people will the population increase in 10 years.
$Annual\ Growth\ (G) = Starting\ Number\ (P) * (1 + amount\ of\ change\ each\ year(r))^{number\ of\ years\ (t)}$
$G = P * (1 + r)^t$
What is my Starting Number *P*?
$886\ million$
What is my amount of change each year?
$1.9\% = 0.019$
What is my equation?
$G = 886{,}000{,}000 * (1 + 0.019)^{10}$

Name: ______________________________

Class: ______________________________

Date: ______________________________

Exponential Functions: Growth and Decay Introduction

General Growth Function:

$$Annual\ Growth\ (G) = Starting\ Number\ (P) * (1 + amount\ of\ change\ each\ year(r))^{number\ of\ years\ (t)}$$

$$G = P * (1 + r)^t$$

General Decay Function:

$$Annual\ Decay\ (D) = Starting\ Number\ (P) * (1 + amount\ of\ change\ each\ year(r))^{number\ of\ years\ (t)}$$

$$D = P * (1 - r)^t$$

Problems

1. Write an exponential growth function to model the situation.

    a. You start with $30,000 and earn 15% interest each year.

    b. Percentage of increase = 15% (0.15)

    c. Number of years = 25

2. The population of Baconburg starts off at 20,000, and grows by 13% each year. Write an exponential growth function and find the population after 10 years.

3. The population of Italy was about 57 million in 1995 and was growing at a rate of 0.4% per year. Write an exponential growth function and find the population after 5 years.

4. The value of a new car declines 9.5% each year over six years. Write an exponential growth function and find the value of a new car which sold for $15,000 after six years.

5. Does the equation $y = 11(1.11)^t$ model exponential growth or exponential decay? Why?

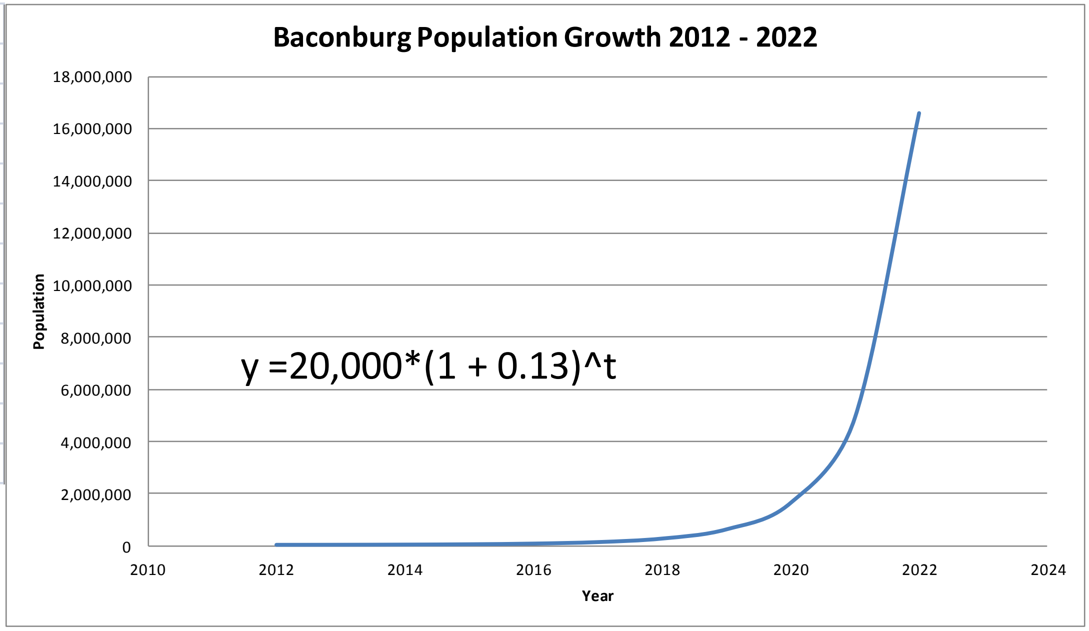
6. Does the equation $y = 7\left(\frac{3}{4}\right)^t$ model exponential growth or exponential decay? Why?

Graphing and Analyzing Exponential Growth / Decay

1. Look at the first question from the problem set.

   The population of Baconburg starts off at 20,000, and grows by 13% each year. Write an exponential growth function and find the population after 10 years.

Year	Population
2012	20,000
2013	22,600
2014	28,858
2015	41,639
2016	67,891
2017	125,085
2018	260,422
2019	612,670
2020	1,628,748
2021	4,892,828
2022	16,609,035



   a. Create two columns: **Year** and **Population**.

   b. In the **Year** column, start with 2012 and add 10 years.

   c. In the **Population** column, opposite 2012, type **20000**. Ensure the cell is formatted so that the number appears as in the table above.

   d. In the **Population** column, opposite **2013**, type the following equation:

$$= 20000 * (1 + 0.13)^\wedge(A3 - 2012)$$

   The value that appears should be 22,600.

   Note: *A3* is the cell that **2013** appears in. If it appears in a different cell on your spreadsheet, type that number instead.

e. Copy and paste to get the remaining population values.

f. Graph the table using a scatter plot and selecting the solid line instead of the individual data points. Label the graph and axes as in the above example.

g. Insert a polynomial trend line and its equation. Retype the equation as shown in the graph above.

Name: ______________________________

Class: _______________________________

Date: _______________________________

Exponential Growth and Decay Exercises

Build tables and graphs in a spreadsheet for the following problems. Attach printouts to this sheet.

1. Estimate the increase in the world's population from 1997 to 2011. Start with the 1997 population of 5.85 billion. Assuming that the population increases by 1.52% each year, the new population each year will be (100 + 1.52%) or (1 + 0.0152) times the previous year's population. So use (1 + 0.0152) as a multiplier.

2. Estimate the increase in Spain's population from 1992 to 2000. Start with the 1992 population of 39 million. Assuming that the population increases by 0.3% each year, the new population each year will be 100 + 0.3% or 1 + 0.003 times the previous year's population. So use (1 + 0.003) as a multiplier.

3. Mike sold an antique car for $32,000. The car's value has been growing at a rate of 7% (0.07) per year for the last 25 years. What was the car valued at 25 years ago?

4. The population of a city in the United States was 154,494 in 2000 and is decreasing at a rate of 1% (0.01) per year. When will the city's population fall below 140,000?

5. In 1992, India had an estimated population of about 886 million people and was growing at a yearly rate of 1.9%. At this rate, by how many people will the population increase in 10 years?

Name: ______________________________

Class: ______________________________

Date: ______________________________

Exponential Growth and Decay Exercises II

Using your EXCEL tables and graphs, answer the following questions:

1. For problem number 1:

   a. Forecast the World Population in 2020: ___________________________

   b. Forecast the year at which the World Population will exceed 10 billion:

      ____

2. For problem number 2:

   a. Forecast the year at which Spain's population will exceed 45 million:

      ____

3. For problem number 3:

   a. If Mike's car's value had been growing at a rate of 5% for the last 25 years, what would its value have been 25 years ago?

      ____

4. For problem number 4:

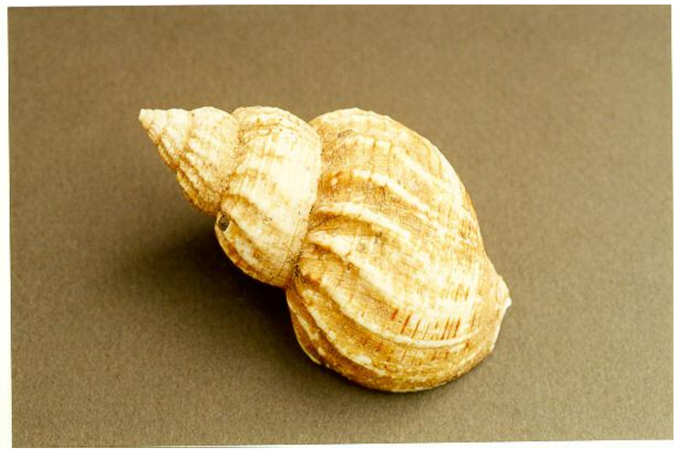
   a. Forecast the year that the city's population will fall below 130,000:

      ____

Name: ____

Whelk – Come to Mathematics[7]

I. **Background.** Sea gulls and crows feed on various types of mollusks by lifting them into the air and dropping them onto a rock to break open their shells. Biologists have observed that northwestern crows consistently drop a type of mollusk called a whelk from a mean height of **about 5 meters**. The crows appear to be selective; they pick up only large-sized whelks. They are also persistent. For instance, one crow was observed to drop a single whelk 20 times. Scientists have suggested that this behavior is an example of decision-making in optimal foraging.

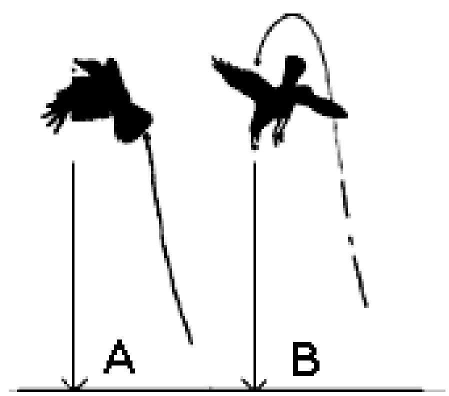


II. **Guiding Question:** Why do you think crows consistently fly to a height of about 5 meters before dropping a whelk onto the rocks below?

III. **Questions.** The figure below left shows the possible flight paths (A or B) of northwestern crows when they are dropping whelks.

A. **Think about this situation and answer the following questions**.

1. Which flight path, A or B, do you think the crows use most? Why?



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[7] From NCTM Illuminations, http://illuminations.nctm.org/LessonDetail.aspx?ID=L480, © 2000 - 2011 National Council of Teachers of Mathematics.

Name: ____

2. What factors do you think influence the height at which the crows choose to drop the whelk?

____

____

____

____

____

3. Do you think there are a minimum or maximum number of drops required to break a whelk?

____

____

____

4. Do you think there is a minimum or maximum height at which a whelk can be dropped to break?

____

____

____

Name: ____

B. What classroom experiment could model the dropping of whelks to collect and analyze data?

C. What questions would you attempt to answer in your experiment?

Name: ____

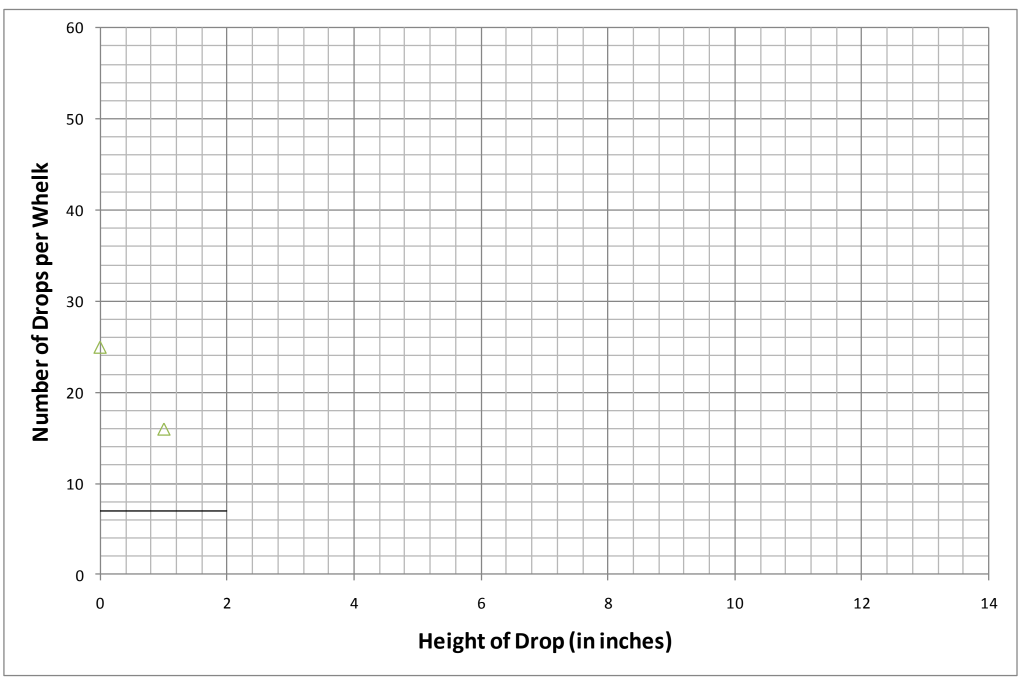
D. How would the relationship between the number of drops and the height of the drops help you answer your questions?

____

____

____

E. Sketch a possible graph of the number of drops required to break a whelk as a function of the height of the drop. How are your answers above evident in your graph?



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Name: ____

Vocabulary Sheet

Mollusk: an invertebrate animal, such as a snail or squid, usually encased in a large shell.

Optimal: most desirable or satisfactory.

Foraging: to collect food or provisions.

Write down any others words you do not know the meaning of:

____

____

____

____

____

____

____

Name: ____

Analyzing the Result of an Experiment[8]

I. **Background.** Sea gulls and crows feed on various types of mollusks by lifting them into the air and dropping them onto a rock to break open their shells. Biologists have observed that northwestern crows consistently drop a type of mollusk called a whelk from an average height of about 5 meters. The crows appear to be selective; they pick up only large-sized whelks. They are also persistent. For instance, one crow was observed to drop a single whelk 20 times. Scientists have suggested that this behavior is an example of decision-making in optimal foraging.

II. **Guiding Question:** Why do you think crows consistently fly to a height of about 5 meters before dropping a whelk onto the rocks below?

III. **The Experiment.** Are the crows minimizing their work by dropping whelks as they do? The amount of work depends upon the height of the drop and the number of times the crow has to fly to this height. To answer the question, the relationship between the height of the drop and the number of drops is needed.

Science students conducted the following experiment. They repeatedly dropped peanuts from a fixed height until the peanut broke. They recorded the height and the number of drops required. They repeated this for several different heights. Their results are printed in the table below:

Height of Drop (in centimeters)

Number of Drops

Peanut Number	15	20	25	30	35	40	50	60
1	15	14	2	11	1	1	2	3
2	21	3	5	1	2	4	2	1
3	30	13	13	4	6	2	1	2
4	6	9	7	4	5	1	1	2
5	8	12	8	5	4	1	4	3
6	24	8	9	3	3	9	5	1
7	19	5	4	6	3	6	4	2
8	15	10	9	7	9	2	2	4
Average								

[8] From NCTM Illuminations, http://illuminations.nctm.org/LessonDetail.aspx?ID=L481, © 2000 - 2011 National Council of Teachers of Mathematics.

Name: ____

IV. **Instructions.**

A. **Enter the table into your spreadsheet.**

B. **For each column of data, compute the average value of the numbers in the column and enter that number in the row called "Average".**

For example, you would compute the average of the first column by adding all the numbers and then dividing them by how many there are:

$$\frac{15 + 21 + 30 + 6 + 8 + 24 + 19 + 15}{8} = 17.25$$

C. **Graph the following data:**

1. Select "Height of Drop" as your *x* value.
2. Select "Average" as your *y* value.
3. Fit a line or curve to the data.
4. Print your table and graph and attach to this sheet.

D. **Answer the following questions:**

1. From your data and graph, what do you think was the best height to drop a peanut to keep the number of drops low but not have to go too high to drop it?

____

Name: ____

2. **In 2 or more sentences,** why do you think this is the best height (*hint:* consider the amount of work you have to do to lift the peanut to a height and the number of drops it takes to break it).

3. From your answer above, what can you say about why crows always drop their whelks from an average height of 5 meters?

Name: ________________________________

Date: ________________________________

Class: ________________________________

Algebra I Practice Assessment I: Exponents and Exponential Functions

Show Work on Separate Sheets of Paper

1. Simplify each product.

a. $(7^2)(7^{-1})$ b. $(8a^3b^4)(4a^3b^2c^5)$ c. $(-f^6g^7h^8)(-f^5g^2h^3)$

$\mathbf{7^{2-1} = 7^1 = 7}$

d. $(7g^5f^8)(-12f^7g^9)$ e. $(8^5)(8^{12})$ f. $(-5x^37y^6)(-6y^4x^2)$

2. Simplify each expression.

a. $(2x^3)^3$ b. $(-x^5)^6$ c. $(-4y^6z^7)^5$

$\mathbf{2^3x^{3*3} = 8x^9}$

d. $(-c^5v^5)^5$ e. $(x^3y^6)^3(-xz)^5$ f. $\left(-qr^5\right)^3(2a^3b^4)^5$

g. $\frac{8^9}{8^3}$ h. $\frac{8^2}{4^3}$ i. $\frac{35k^5}{7k^3}$

$\mathbf{8^{9-3} = 8^6}$

j. $\left(\frac{12c^5d^6}{3c^3d^2}\right)^3$ k. $\frac{(x^3y^4z^2)^2}{x^2y^4z^2}$ l. $\frac{(7x^4z^6)^2}{6x^2y^3}$

3. Write each expression without negative or zero exponents.

a. 6^{-3} b. -10^3 c. -6^3

$\mathbf{\frac{1}{6^3}}$

d. r^3s^{-2} e. x^0d^{-1} f. $-a^2b^{-3}$

g. $\frac{x^4}{y^{-2}}$ h. $\frac{(2x^3)(10y^5)}{5y^{-3}}$ i. $\frac{g^{-2}h^{-3}}{i^{-2}j^{-3}}$

4. Write each number in scientific notation.

a. 5,200,000,000 b. 0.0000063 c. 8,430,000

$\mathbf{5 * 10^9}$

5. Write each number in decimal notation.

a. 5.95×10^6 b. 6.92×10^{-3} c. -8.5×10^4

$\mathbf{595,000}$

6. Simplify. Write your answers in scientific notation.

a. $(7 \times 10^3)(8 \times 10^5)$ b. $\frac{8\times10^5}{7\times10^3}$ c. $5 \times 10^5 - 3 \times 10^5$

$\mathbf{(7 * 8) * \left(10^3 * 10^5\right)}$

$\mathbf{56 * 10^{3+5}}$

$\mathbf{56 * 10^8}$

7. Answer the following:

    a. Does the equation $y = 6 * (1 + 0.03)^t$ model exponential growth or exponential decay? Why?

    b. The population of the state of Rhode Island's population has been declining each year from 1,080,632 persons in 2004 at an average rate of 1%. Write an equation for the decline in Rhode Island's population.

Name: ___________________________

Date: ___________________________

Class: ___________________________

Algebra I: Exponents and Exponential Functions
Show Work on Separate Sheets of Paper

1. Simplify each product.

a. $(8^2)(8^{-1})$ b. $(7v^3w^4)(6v^3w^2)$ c. $(-8t^47u^3)(-9t^4u^2)$

2. Simplify each expression.

a. $(3d^4)^3$ b. $(-n^5)^8$ c. $(-12o^{11}p^7)^6$

d. $\frac{5^7}{5^2}$ e. $\frac{9^7}{3^6}$ f. $\frac{64t^5}{8t^3}$

g. $\frac{24c^8d^{12}}{4c^6d^6}$ k. $\frac{(b^5c^9d^3)^2}{b^2c^4d^2}$

3. Write each expression without negative or zero exponents.

a. 12^{-5} b. -2^6 c. . c^0b^{-6}

d. $\frac{t^7}{g^{-7}}$ e. $\frac{(4u^4)(19v^8)}{w^{-2}}$ f. $\frac{b^{-1}v^{-12}}{c^{-5}w^{-6}}$

4. Write each number in scientific notation.

   a. 8,700,000   b. 0.000000095   c. 128,789,000

5. Write each number in decimal notation.

   a. 9.3×10^7 b. -7.345×10^7

6. Simplify. Write your answers in scientific notation.

   a. $(6 \times 10^5)(9 \times 10^6)$ b. $12 \times 10^7 - 8 \times 10^7$

7. Answer the following:

   a. Does the equation $y = 8 * (1 - 0.14)^t$ model exponential growth or exponential decay? Why?

   b. The population of the Klingon homeworld, Q'nos, has been declining since its moon, Praxis, exploded on star date 9521.6. The **original population was 6,000,000,000** and has been declining at a rate of **5% per year**. Starfleet Command requests the Captain of the U.S.S. *Enterprise* (NCC – 1701A) develop an equation to determine the population that will need to be evacuated from Q'nos if the Federation can start the evacuation **5 years** after the destruction of Praxis. **What is the equation?**

PAUL CUFFEE SCHOOL

A Maritime Charter School for Providence Youth

Congruence and Similarity

9th Grade Mathematics

Captain Thomas R. Beall, U. S. Navy (Ret.)

Introduction to Congruence and Similarity

I. **Day 1: Review of Where We Have Been and Introduction to Polygon Congruence.**

A. *Warm-up.*

1. Find the surface area and volume of a triangular prism with triangle base, *b*, equal to 5 in., triangle height, *h*, equal to 4.5 in., and prism height, *H*, equal to 20 in.

2. Find the volume of a cone with base diameter of 20 cm., and height 35 cm.

B. *Quiz Review.* Briefly review answers to quiz from attached key.

C. *Review* the basic formulas for surface area and volume of 3D shapes using summary sheet below. Ensure students have the formulas listed in their "little books".

D. Now that we understand these basic shapes, we will consider how we might use them. We have already discussed cargo movement. We will now consider other concepts.

1. Consider the small boat model we build at CBC before we started the bigger boat. What was the relationship between the parts of the model and the parts of the real boat?

2. Consider the model rockets you are working on in Ms. Cullen's class. What is the relationship between such a model rocket and a real one such as you might see in this video?

RENTZ Missile Shoot Nov 02.mpg

3. We've talked about scaling of basic shapes – how a scale model relates to the real thing and vice versa. What defines the relationship between the two?

4. To establish a relationship, we need to consider the concepts of congruence and similarity.

Polygon Congruence Postulate. Two polygons are congruent if and only if there is a correspondence between their sides, such that:

1. Each pair of corresponding angles is congruent.
2. Each pair of corresponding sides is congruent.

E. *Homework.* Holt, *Geometry*, pp. 214 – 215, nrs. 14 – 28.

II. **Day 2: Angle Measures in Polygons.**

A. Warm-up. Students should do this as they come in. Normally you will have to get them settled down and started.

1. Find the area of a regular hexagon with side length of 10 in. and apothem 7.5 in. The answer is $\frac{1}{2}(10 \times 7.5) \times 6 = 225\ in.^2$.

2. Find the volume of a hexagonal prism with the base area of the hexagon above and a height of 20 in. The answer is $225\ in.^2 \times 20\ in. = 4500\ in.^3$.

B. Intro: Ask students to consider a triangle. Ask them, "What is the sum of the angles of a triangle?

C. *Activity 1*: Measuring interior angles.

1. Hand out the activity sheets (ask Elexus or another student to do this). Ask another student to hand out protractors (on my desk).

2. Ask students to use their protractors to measure the interior angles of each triangle on the first page.

a. Although we have used protractors before, you may need to offer help. Encourage members of the group to help each other – they should be able to recall how to use the protractors.
b. The angles are all less than 90°.
c. The interior angles of each triangle should add up to 180°. Students will likely not come up with exactly 180° but they should come close.

Triangle Sum Theorem:

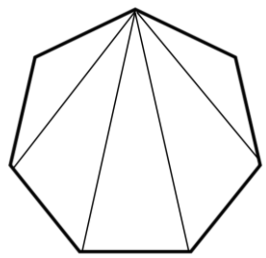
The sum of the measures of a triangle is 180°.

D. *Activity 2*: Angle Measures in Polygons.

1. Now that students have concluded that the sum of the interior angles of a triangle add up to 180° they will explore what the angles of other polygons add up to.

2. Activity involves subdividing the polygons into triangles and then realizing that, if each triangle's angles add up to 180° then the angles of the polygon should be 180° times the number of triangles in the polygon.

3. While instructions seem pretty clear, you will have to model how to divide a polygon into triangles. Using an overhead (provided) and the overhead projector, subdivide the heptagon (seven-sided figure) like the one below – highlighting that the triangles should be drawn from only one vertex.



E. *Activity 3*: Filling out the Table. Students should now move on to filling out the table on the next page. Answers to questions follow:

- Question 2: If the sum of the interior angles of one triangle is 180 degrees, how could you determine the sum of the interior angles of any convex polygon?

  Answer: Find the number of triangles the polygon can be divided into and then multiply that number by 180°.

- Question 3: Fill in the table below. It will look like this:

Name of Polygon	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon	10-gon	n-gon
Number of Sides	3	4	5	6	7	8	10	
Number of Triangles	1	2	3	4	5	6	7	n-2
Sum of Interior Angles	180°	360°	540°	720°	900°	1080°	1260°	(n-2)*180

- Question 4: In words, if a polygon has n sides:

a. How many triangles can you draw? Ans.: n – 2.
b. What is the sum of the interior angles? Ans.: $(n-2)\times 180°$.

- Write an equation for the sum, *S*, of the interior angles of a polygon. ***This is a definition that students should put into their notes.***

$$S = (n-2)\times 180°$$

F. *Activity 4*: Homework. Worksheet package includes copies of pages from Holt, *Geometry*. Students should do the following:

p. 173, nrs. 8 – 12.
p. 180, nrs. 9 – 13, odd.
p. 181, 19-21, 25-31.

You may have to model how to do 25 – 31. Both figures are quadrilaterals and therefore the sum of the angles is 360°. For the first one, x=36°. You should be able to determine the angle measures now that you know the value of x.

$$4x + 3x + x + 2x = 360° = 10x, x = 36°$$

III. **Day 3: Triangle Congruence.**

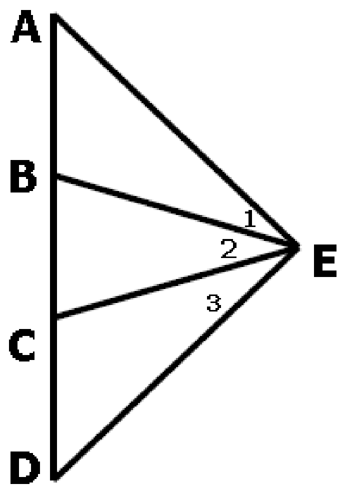
A. Warm-up.

B. Congruence.

1. Activity 1: Go over the postulate definitions with students and have them draw the congruent triangles.

2. Activity 2: Students prove that triangles are congruent using the postulates.

Proof THREE



Given :

$\measuredangle 1 \cong \measuredangle 2$

$\measuredangle 2 \cong \measuredangle 3$

$\overline{EB} \cong \overline{EC}$

$\overline{AE} \cong \overline{DE}$

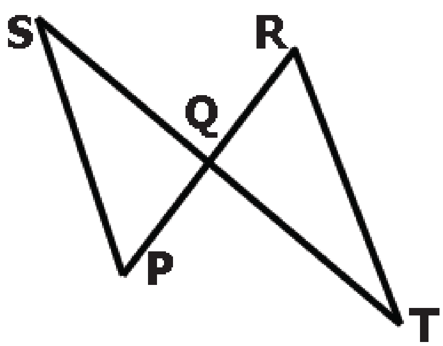
Prove:

$\triangle AEB \cong \triangle DEC$

$\measuredangle 1 \cong \measuredangle 2$	Given
$\measuredangle 2 \cong \measuredangle 3$	Given
$\measuredangle 1 \cong \measuredangle 3$	Transitive Property
$\overline{EB} \cong \overline{EC}$	Given
$\overline{AE} \cong \overline{DE}$	Given
$\triangle AEB \cong \triangle DEC$	SAS

A. **ASA Postulate.**

Proof SIX



Given:

Q is the midpoint of $\overline{PR}$

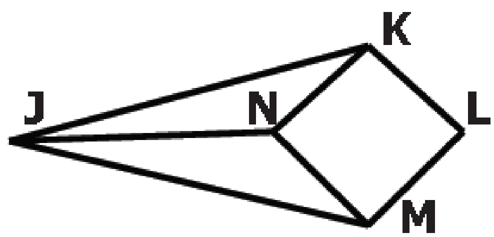
$\measuredangle P \cong \measuredangle R$

Prove:

$\triangle SPQ \cong \triangle TRQ$

$\angle P \cong \angle R$	Given
Q is the midpoint of $\overline{PR}$	Given
$\overline{PQ} \cong \overline{QT}$	Definition of midpoint
$\angle SQP \cong \angle TQR$	Vertical angles are congruent
$\Delta SPQ \cong \Delta TRQ$	ASA Postulate

Proof FIVE



Given:

□NKLM is a square

$\overline{JK} \cong \overline{JM}$

Prove:

$\triangle JKN \cong \triangle JMN$

□NKLM is a square	Given
$\overline{NM} \cong \overline{NK}$	All sides of a □ are $\cong$
$\overline{JN} \cong \overline{JN}$	Reflexive Property
$\overline{JK} \cong \overline{JM}$	Given
$\triangle JKN \cong \triangle JMN$	SSS

3. Activity 3: Homework Worksheet. Holt, *Geometry*, pp. 221 – 222, 8 – 19, 23 – 29.

IV. **Day 4: Quiz Review – Game.**

A. Warm-up. Posted in classroom already. Two equations problems from SAT study guide. Students are in circle during warm-up.

  A. During warm-up, have students pass in homework for check. Pass it back when checked. This is a new process which may be a little rocky. If TP is assisting, ask TP to walk around check WU progress while I check off homework.

B. Homework Review. This will be done at overhead projector. Students are in circle during homework review.

  A. Do quick check with students (thumbs up or down) to see assess how they feel about HW.

  B. Ask students if they have any problems they would like solved or with which they had difficulty.

C. Quiz Review. Jeapordy. Students are in groups.

  A. Rules:

    1. I pick a number between 1 and 20. Group closest gets to pick first. Rotation then works like Jeapordy game – group getting a correct answer gets to pick a new category.

    2. Groups compete. After a category is revealed, first group to get an answer raises hand (not calling out). I pick the group and ask for the answer. If answer is correct, that group gets to pick again. If incorrect, I await another group (the group that answered incorrectly can try again).

Answer Key

	Prisms	Cylinders	Cones	Angle Measure and Congruence
10	Find surface area of a rectangular prism with dimensions 5 in. x 3 in. x 10 in.	Find the volume of a cylinder A cylinder with radius 25 in. and height 100 in.	Find the volume of a cone with radius 25 in. and height 75 in. (49087.4 in^2, 15625 pi in^2)	State the polygon congruence postulate. Two polygons are congruent if and only if there is a correspondence between their sides, such that: Each pair of corresponding angles is congruent. Each pair of corresponding sides is congruent.
20	Find the surface area of a triangular prism triangular pyramid of b = 3 in., h = 2.6 in., H = 6 in., sh (slant height) = 6.3 in.	Find the surface area of a cylinder with diameter of 40 in. and height of 80 in.	Find the volume of a cone with diameter 49 cm. and height of 66 cm.	State the SAS postulate.
30	Find the area of a regular hexagon with side length S = 8 m and apothem a = 6.8 m.	Find the volume of a cylinder with diameter of 28 cm. and a height of 134 cm.	Find the surface area of a cone with radius of 2 ft. and height of 10 ft.	What is the measure of an interior angle of an equilateral hexagon?
40	Find the volume of a right regular hexagonal prism with side length S = 10 m, apothem a = 8.3 m and height H = 20 m.	Find the surface area **and** volume of a cylinder with diameter of 30 ft. and height of 110 ft.	Find the surface area of a cone with diameter of 22 in. and height of 50 in.	What is the sum of the measures of the interior angles of an octagon?
50	Find the surface area of a right regular hexagonal prism with side length S = 10 m, apothem a = 8.3 m and height H = 20 m.	Find the surface area **and** volume of a cylinder with diameter of 43 ft. and a height of 78 ft.	Find the surface area ***and*** volume of a cone with diameter of 23 ft. and height of 72 ft.	An interior angle measure of a regular polygon is 165°. Find the number of sides of the polygon.

Prisms	Cylinders	Cones	Angle Measure and Congruence
$10	$10	$10	$10
$20	$20	$20	$20
$30	$30	$30	$30
$40	$40	$40	$40
$50	$50	$50	$50

Group Scores:

Group 1:

Group 2:

Group 3:

Group 4:

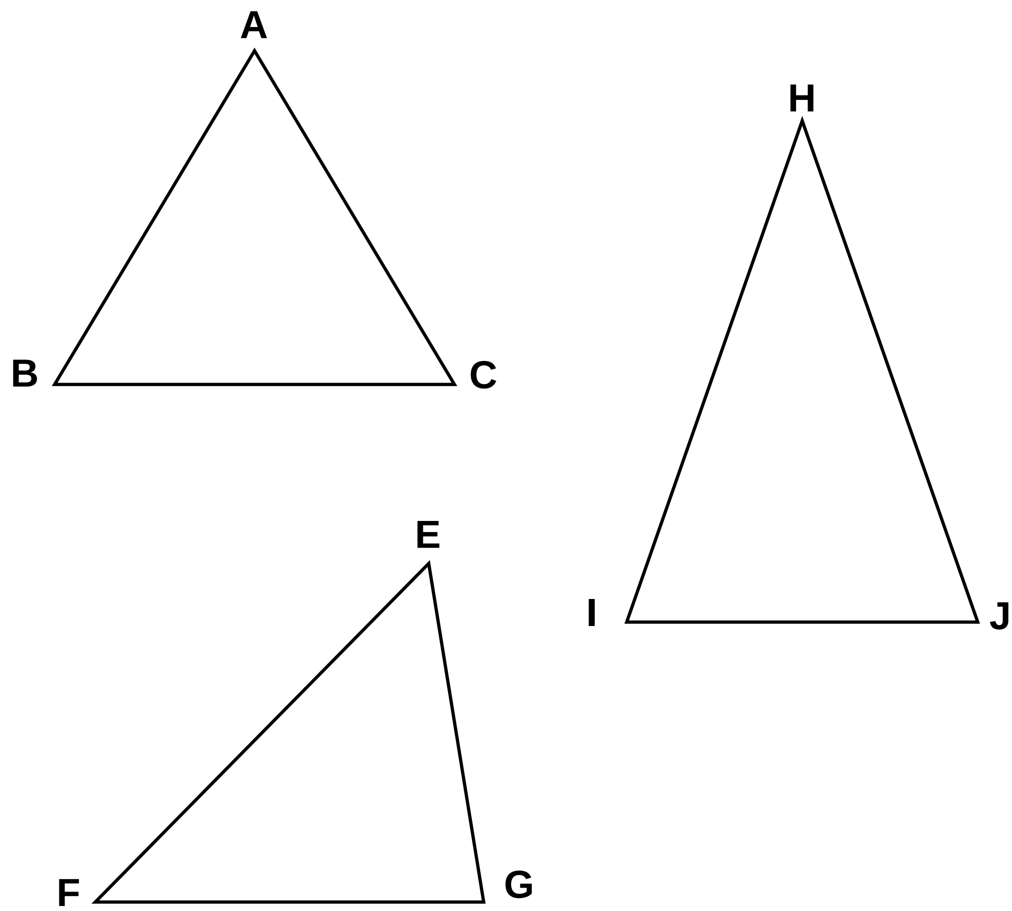
Group 5:

Group 6:

D. In-class worksheet. Time permitting, students will work on the following: Holt, *Geometry*. This will not be assigned as homework.

Sum of the Interior Angles of a Triangle

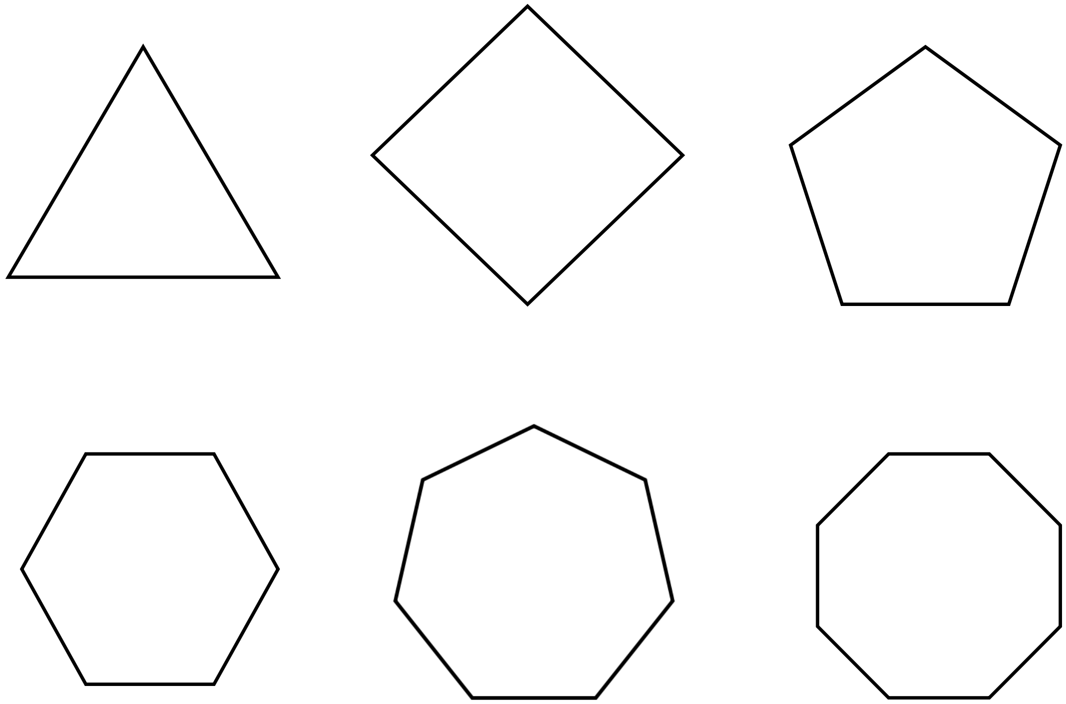
1. Using a protractor, measure all three interior angles of each triangle. Add the three angles of each triangle. What to they add up to?



2. What conjecture can you make about the sum of interior angles of any triangle?

Angle Measures in Polygons

1. In each of the polygons below, from one and only one vertex, draw diagonals to divide the polygon into as many triangles as you can.



2. If the sum of the interior angles of one triangle is 180 degrees, how could you determine the sum of the interior angles of any polygon?

3. Fill in the table below.

Name of Polygon	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon	10-gon	n-gon
Number of Sides								**n**
Number of Triangles								
Sum of Interior Angles								

4. In words, if a polygon has n sides:
   a. How many triangles can you draw?

   a. What is the sum of the interior angles?

5. Write an equation for the sum, S, of the interior angles of a polygon.

S =

Adapted from *Illuminations* "Adding It All Up" Activity Sheet, http://illuminations.nctm.org/Lessons/AddingItAllUp/AddingItAllUp-AS.pdf

Name: ________________________

Class:__________________________

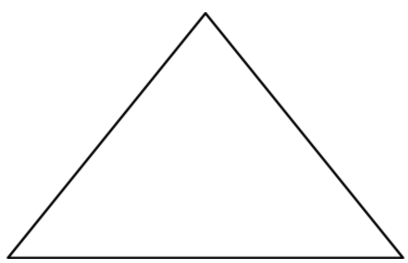
Date:__________________________

Triangle Congruence – November 9th, 2010

I. Congruence Postulates:

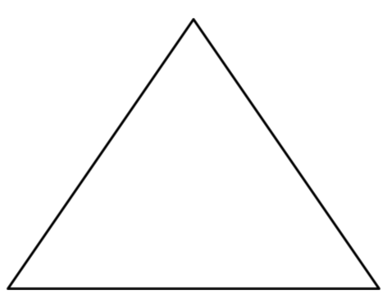
A. **SSS (Side-Side-Side Postulate):** If the ______________________ of one triangle are congruent to the __________________________ of another triangle, then the triangles are congruent.

Draw and label a triangle that is congruent to this one by the SSS postulate:



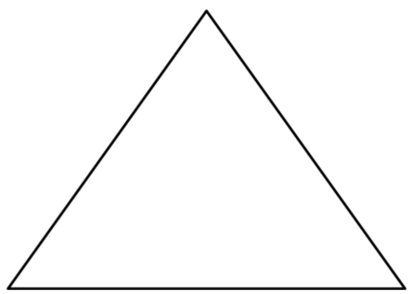
B. **SAS (Side-Angle-Side Postulate):** If two __________________________ and their ____ in one triangle are congruent to __________________________ and their ____ in another triangle, then the two triangles are ____________________________________.

Draw and label a triangle that is congruent to this one by the SSS postulate:



C. **ASA (Angle-Side-Angle) Postulate:** If two ______________________________ and the corresponding _______________________ in one triangle are congruent to two _______________________________ and the corresponding ______________________ in another triangle, then the two triangles are ___________________________________.

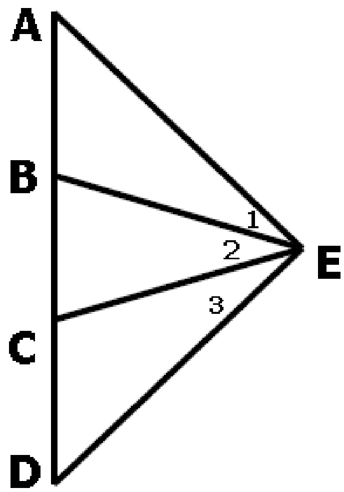
Draw and label a triangle that is congruent to this one by the SSS postulate:



II. **Proving Triangles are Congruent Using the Postulates.**

B. **SAS Postulate.**

Proof THREE



Given :

$\measuredangle 1 \cong \measuredangle 2$

$\measuredangle 2 \cong \measuredangle 3$

$\overline{EB} \cong \overline{EC}$

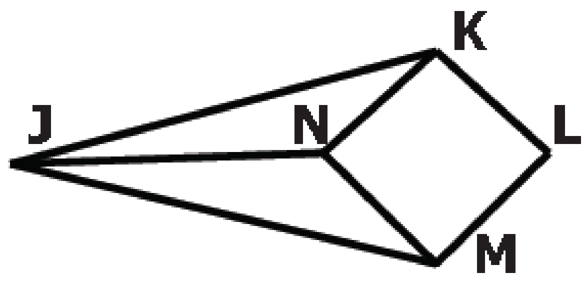
$\overline{AE} \cong \overline{DE}$

Prove:

$\triangle AEB \cong \triangle DEC$

C. **SSS Postulate.**

Proof FIVE



Given:

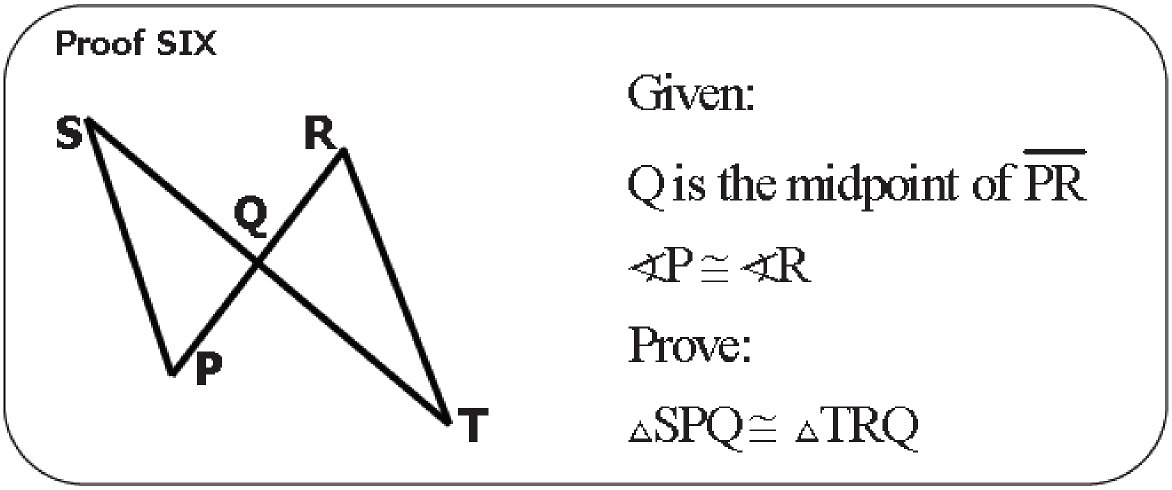
□NKLM is a square

$\overline{JK} \cong \overline{JM}$

Prove:

$\triangle JKN \cong \triangle JMN$

D. **ASA Postulate.**



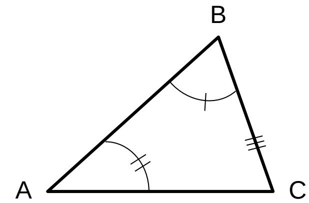
$\angle P \cong \angle R$	Given
Q is the midpoint of $\overline{PR}$	Given
$\overline{PQ} \cong \overline{QT}$	Definition of midpoint
$\angle SQP \cong \angle TQR$	Vertical angles are congruent
$\Delta SPQ \cong \Delta TRQ$	ASA Postulate

Proof of AAS Congruency Theorem:

If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.

Geometry Review Nr. 1 – Surface Area & Volume of 3D Shapes / Logic and Proof / Triangle Congruence

I. **Logic and Proof.**

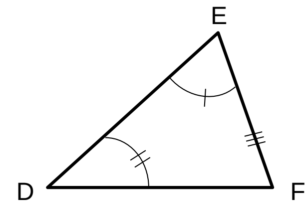


Given:

$\angle A \cong \angle D, \angle B \cong \angle E, \overline{BC} \cong \overline{EF}$

Prove:

$\Delta ABC \cong \Delta DEF$



$\angle A \cong \angle D$	Given
$\angle B \cong \angle E$	Given
$\overline{BC} \cong \overline{EF}$	Given
$m\angle A + m\angle B + m\angle C = 180°$	Triangle Sum Theorem
$m\angle D + m\angle E + m\angle F = 180°$	Triangle Sum Theorem
$m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$	Transitive Property
$m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$	Substitution Property
$m\angle C = m\angle F$	Subtraction Property
$\Delta ABC \cong \Delta DEF$	ASA Postulate

II. **Surface Area & Volume of 3D Shapes.** Find the area, surface area, and / or volume of the given shapes.

A. A triangular prism of $S_1 = S_2 = S_3 = b = 5$ in., $h = 4.3$ in., $H = 8$ in.

1. Surface Area: 141.5 in.^{2}
2. Volume: 86 in.^{3}

B. A cylinder with diameter 22 in. and height 65 in.

1. Surface Area: 1672 pi. in.^{2} / 5250.08 in.^{2}
2. Volume: 7865 pi. in.^{3} / 24696.1 in.^{3}

C. A right regular hexagonal prism with side length $S = 12$ m, apothem $a = 10.6$ m and height $H = 133$ m.

1. Surface Area: 10339.2 m.^{2}
2. Volume: 50752.8 m.^{3}

III. **Triangle Congruence.**

A. SAS (Side-Angle-Side) Postulate – State the SAS Postulate.

If two sides and their included angle in one triangle are congruent to the corresponding sides and their included angle in another triangle, then the two triangles are congruent.

B. Draw a second triangle and label both to show they are congruent by the ASA (Angle-Side-Angle) Postulate:

C. What is the sum of the interior angles of a 12-gon (a 12 sided figure)?

$$(12-2)\times 180° = 1800°$$

D. An interior angle measure of a regular polygon is 146°. Find the number of sides of the polygon.

$$146° = 180° - \frac{360°}{n}$$
$$-36 = \frac{-360°}{n}$$
$$n = \frac{360°}{36} = 10$$

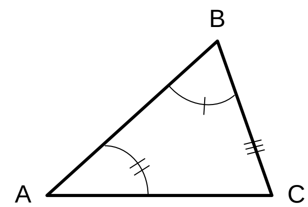
Name: ___________________________

Class:____________________________

Date:____________________________

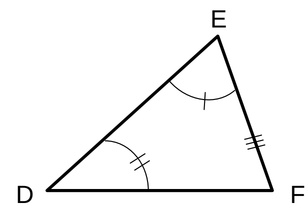
Geometry Review Nr. 1 – Surface Area & Volume of 3D Shapes / Logic and Proof / Triangle Congruence

I. **Logic and Proof.**



Given:

$\angle A \cong \angle D, \angle B \cong \angle E, \overline{BC} \cong \overline{EF}$



Prove:

$\Delta ABC \cong \Delta DEF$

II. **Surface Area & Volume of 3D Shapes.** Find the area, surface area, and / or volume of the given shapes.

A. A triangular prism of $S_1 = S_2 = S_3 = b = 5$ in., $h = 4.3$ in., $H = 8$ in.

1. Surface Area:

2. Volume:

B. A cylinder with diameter 22 in. and height 65 in.

1. Surface Area:

2. Volume:

C. A right regular hexagonal prism with side length $S = 12$ m, apothem $a = 10.6$ m and height $H = 133$ m.

1. Surface Area:

2. Volume:

III. **Triangle Congruence.**

A. SAS (Side-Angle-Side) Postulate – State the SAS Postulate.

B. Draw a second triangle and label both to show they are congruent by the ASA (Angle-Side-Angle) Postulate:

C. What is the sum of the interior angles of a 12-gon (a 12 sided figure)?

D. An interior angle measure of a regular polygon is 146°. Find the number of sides of the polygon.

Day 3: Analyzing and Using Triangle Congruence

I. Warm-up (10 minutes).

II. **Homework Review (10 minutes).** In the circle we will review the homework. Ask for a thumbs up or down.

III. Activities.

A. **Activity 1: Flowchart Proof (5 minutes).** In the circle, hand out the flowchart proof page, p. 236, of the text and walk thru the concept with students.

B. **Activity 2: Some theorems and corollaries (5 minutes).** In the circle, we will fill in the notetaking sheet with these theorems / corollaries.

1. Isosceles Triangle Theorem. If two sides of a triangle are congruent, then the angles opposite those sides are congruent. Draw a picture illustrating this.

2. Converse of the Isosceles Triangle Theorem. If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

3. Corollary of a Theorem: an additional theorem that can easily be derived from the original theorem.

4. Corollary: The measure of each angle of an equilateral triangle is 60°.

5. Corollary: The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base. Draw a picture illustrating this.

C. **Activity 3 (15 minutes): Prove or disprove the following** conjecture: If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.

Prove or disprove this conjecture (you may draw a picture to demonstrate your proof).

*This theorem (**called the AAS Theorem**) can be proven using the ASA postulate and the fact that the sum of the interior angles of a triangle = 180°.*

This is a good demonstration of how postulates are used to prove theorems.

D. **Activity 4: Practice.** Holt *Geometry*, pp. 240 – 241, 10 – 17 and 24 – 29.

Day 4: Analyzing Triangle Congruence Continued.

I. **Warm-up (10 minutes).**

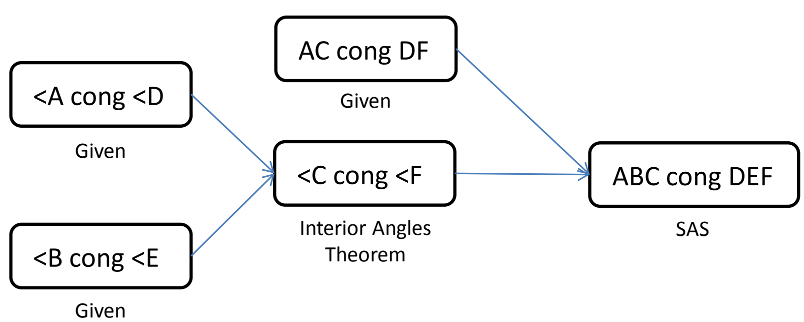
II. **Homework Review (5 minutes).**

III. **Activity 1: Flowchart Proof (10 minutes).** In the circle, hand out the flowchart proof page, p. 236, of the text and walk thru the concept with students.

IV. **Activity 2: Flowchart Proof of AAS Conjecture (10 minutes).** In groups, students will prove the conjecture.

If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.

Draw two triangles that illustrate the conjecture and then prove it using the flowchart proof technique



V. **Activity 3: Guided Skills Practice: Flowchart Proofs, Holt *Geometry*, page 239 (5 minutes).**

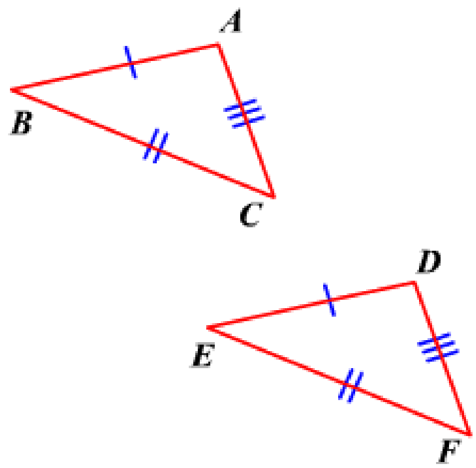
VI. **Activity 4: On large pieces of paper with markers, create flowchart proofs for problems 18 and 19 of Holt, *Geometry*, p. 240 (15 minutes).**

VII. **Activity 5: Holt, *Geometry*, pp. 240 – 241 10-17, 24-29.**

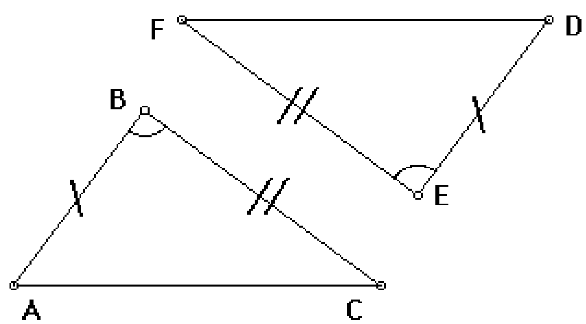
Worksheet – Analyzing Triangle Congruence

1. **Review (5 minutes):** Recall our three triangle congruence postulates:

A. SSS:



B. SAS:



C. ASA:

2. **Activity 1: Invalid Combinations (15 minutes).** In the most recent quiz a number of students drew pictures with invalid combinations. We will look at these and demonstrate why they are invalid.

   A. Conjecture: If three angles of one triangle are congruent to three angles of another triangle, then the triangles are congruent.

      Prove or disprove this conjecture (you may draw a picture to demonstrate your proof).

      Is there a relationship between these triangles?

   B. Conjecture: If two sides and a non-included angle of two triangles are congruent then the triangles are congruent.

      Prove or disprove this conjecture (you may draw a picture to demonstrate your proof).

3. **Activity 2 (15 minutes): Triangle Congruence Theorems.**

   A. Conjecture: If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and leg of another right triangle, then the two triangles are congruent.

      Prove or disprove this conjecture (you may draw a picture to demonstrate your proof). *Hint: Use a concept involving right triangles that you have learned before.*

   B. Conjecture: If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.

      Prove or disprove this conjecture (you may draw a picture to demonstrate your proof).

4. **Activity 3: Practice (10 minutes).** Holt, *Geometry*, p. 231, nrs. 10 – 29, 35.

Triangle Congruence = Some More Definitions, Theorems, and Proofs

1. Isosceles Triangle Theorem. If two _______________ of a triangle are congruent, then the _______________________________________ are congruent. Draw a picture illustrating this.

2. Converse of the Isosceles Triangle Theorem. If two ____________________ of a triangle are congruent, ____ angles are congruent.

3. Corollary of a Theorem: ____
____

4. Corollary: The _______________________________ of an equilateral triangle is 60°.

5. Corollary: The bisector of the vertex angle of an isosceles triangle is ____. Draw a picture illustrating this.

6. **Prove or disprove the following** conjecture: If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.

   Prove or disprove this conjecture (you may draw a picture to demonstrate your proof).

7. **Activity 4: Practice.** Holt *Geometry*, pp. 240 – 241, 10 – 17 and 24 – 29.

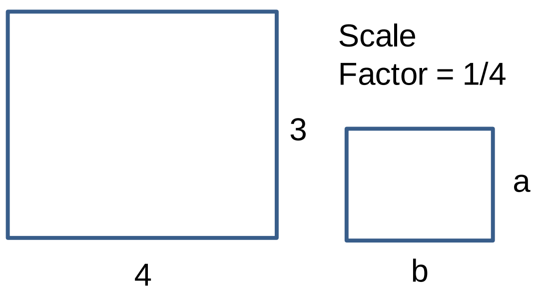
Name: ______________________________

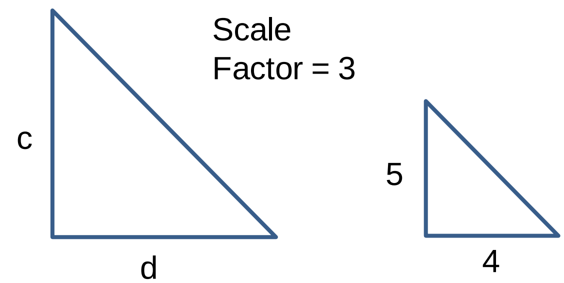
Class: ______________________________

Date: ______________________________

Triangle Similarity Review
Show work on graph paper and separate sheets of paper

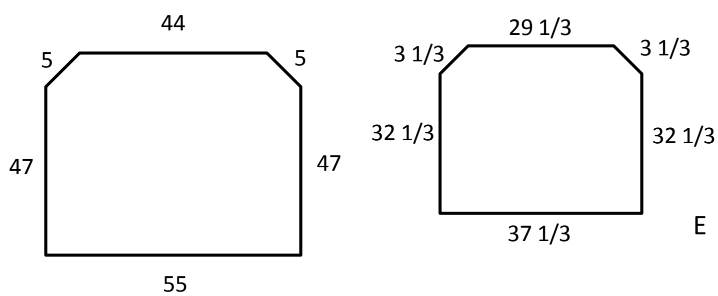
1. **Scale Factors.** Each pair of shapes is similar. Use the given scale factor to find the dimensions of the corresponding sides.

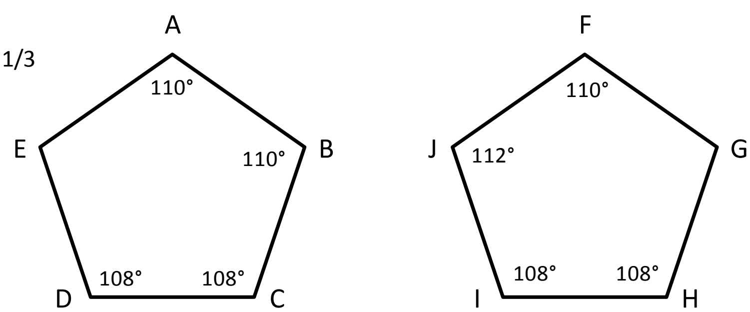




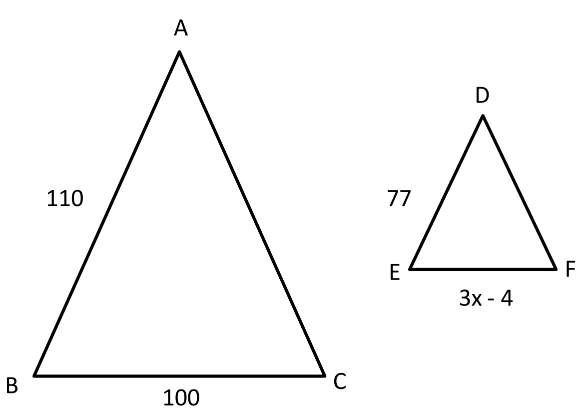
2. **Determining Similarity.** For the given polygons:

- State whether they are similar.
- If they are, state what postulate or theorem supports your conclusion.
- If not, state why not.
- Justify your conclusion by indicating corresponding angles and / showing proportionality of the sides.

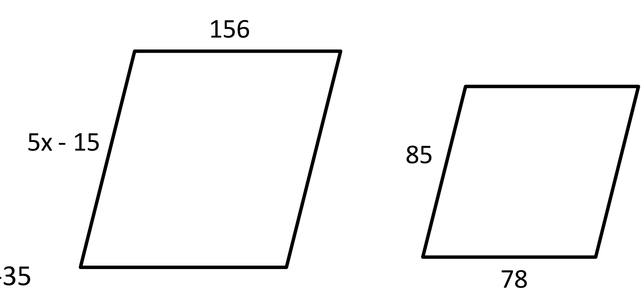




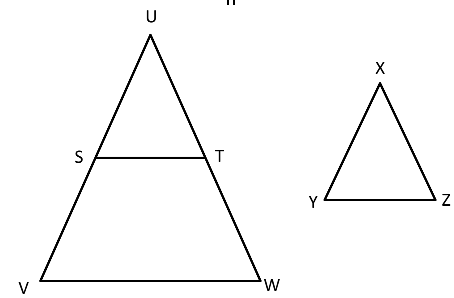
3. In the figures below, the polygons are similar. Find the value of *x*.



C & S-35



4. **Proof.** In the diagram at right, two sides of ΔUVW are proportional to two sides of ΔXYZ, and $\frac{UV}{XY} = \frac{UW}{XZ}$ and $\angle U \cong \angle X$. Also, ST has been added such that US = XY and ST || VW.



Previously, you used the AA Similarity Postulate to prove that ΔUST~ΔUVW.

ST || VW	Given.
$\angle S \cong \angle V$	If two parallel lines are cut by a transversal then the corresponding angles are congruent.
$\angle U \cong \angle U$	Reflexive Property.
ΔUST~ΔUVW	AA Postulate.

Use this proof to prove that ΔUST~ΔXYZ.

Name: ______________________________

Date: ______________________________

Advisor: ______________________________

Developing Similar Shapes Review

1. **Geometry of the Ship Unit.** A shipping container has the following dimensions:

$$20' \times 8' \times 8'$$

You wish to construct a model whose length is on greater than 0.2 feet.

a. What is the scale factor you will need to apply to the dimensions of the actual shipping container to find the dimensions of your model?

Scale factor is: ____

b. What are the dimensions of the model container box?

Dimensions: ____

2. **Geometry of the Ship Unit.** You wish to construct a model of a battleship which has the following dimensions:

Length overall:	680 ft., 9.813 in.
Maximum beam:	108 ft., 2.250 in.
Maximum draft:	36 ft., 9.000 in.

a. If you want your model's length overall to be no more than 36 inches (3 feet), what scale factor will you need to apply to the actual ship dimensions to find the dimensions of your model?

Scale factor is: ____

b. What are the dimensions of the model battleship?

Length overall: ___________________

Maximum beam: ___________________

Maximum draft: ___________________

3. **Navigation Unit.** The distance from the island of O'ahu in Hawai'i to the island of Midway in the Central Pacific Ocean is 1300 nautical miles. On a game board, the distance is 13 feet. What is the scale factor used in the chart (***remember to convert nautical miles to yards and yards to feet or vice versa***).

   Scale factor is: ____

4. **Congruence and Similarity Unit.** You wish to construct a model of the Solar System that is no greater than 25 feet in length with the Sun at one end and Pluto at the other. Complete the following table to determine the distances of each model planet from the model Sun:

Body	**Planet Distance from Sun (km)**	**Scale Factor**	**Model Distance (m)**
Sun			
Mercury	57,950,000		
Venus	108,110,000		
Earth	149,570,000		
Mars	227,840,000		
Jupiter	778,140,000		
Saturn	1,427,000,000		
Uranus	2,870,300,000		
Neptune	4,499,900,000		
Pluto	5,913,000,000		

Name: ______________________________

Date: ______________________________

Advisor: ______________________________

Developing Similar Shapes Review

1. **Geometry of the Ship Unit.** A shipping container has the following dimensions:

$$20' \times 8' \times 8'$$

You wish to construct a model whose length is on greater than 0.2 feet.

a. What is the scale factor you will need to apply to the dimensions of the actual shipping container to find the dimensions of your model?

Scale factor is: **0.01**

b. What are the dimensions of the model container box?

Dimensions: $(\mathbf{20' * 0.01}) \times (\mathbf{8' * 0.01}) \times (\mathbf{8' * 0.01}) = \mathbf{0.2'} \times \mathbf{0.08'} \times \mathbf{0.08'}$

2. **Geometry of the Ship Unit.** You wish to construct a model of a battleship which has the following dimensions:

Length overall:	680 ft., 9.813 in.
Maximum beam:	108 ft., 2.250 in.
Maximum draft:	36 ft., 9.000 in.

a. If you want your model's length overall to be no more than 36 inches (3 feet), what scale factor will you need to apply to the actual ship dimensions to find the dimensions of your model (**remember to convert feet to inches or inches to feet)**?

Scale factor is: **0.0044**

b. What are the dimensions of the model battleship?

Length overall: **36 inches**

Maximum beam: **5.71 inches**

Maximum draft: **1.94**

3. **Navigation Unit.** The distance from the island of O'ahu in Hawai'i to the island of Midway in the Central Pacific Ocean is 1300 nautical miles. On a game board, the distance is 13 feet. What is the scale factor used in the chart (***remember to convert nautical miles to yards and yards to feet or vice versa***).

   Scale factor is: $\mathbf{1.67 \times 10^{-6}}$

4. **Congruence and Similarity Unit.** You wish to construct a model of the Solar System that is no greater than 25 feet in length with the Sun at one end and Pluto at the other. Complete the following table to determine the distances of each model planet from the model Sun:

Body	Planet Distance from Sun (km)	Scale Factor	Model Distance (m)
Sun			
Mercury	57,950,000	0.000000004227972	0.25
Venus	108,110,000	0.000000004227972	0.46
Earth	149,570,000	0.000000004227972	0.63
Mars	227,840,000	0.000000004227972	0.96
Jupiter	778,140,000	0.000000004227972	3.29
Saturn	1,427,000,000	0.000000004227972	6.03
Uranus	2,870,300,000	0.000000004227972	12.14
Neptune	4,499,900,000	0.000000004227972	19.03
Pluto	5,913,000,000	0.000000004227972	25

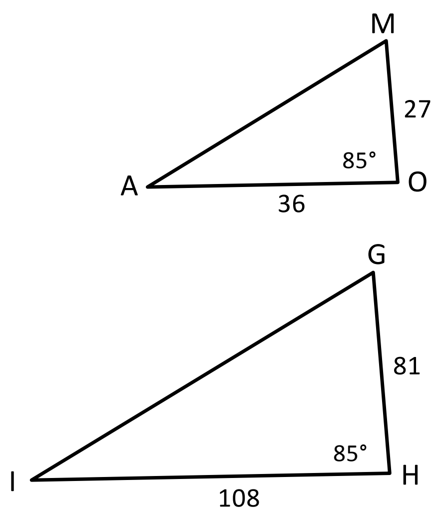
Congruence and Similarity Practice I
Show work on graph paper and separate sheets of paper

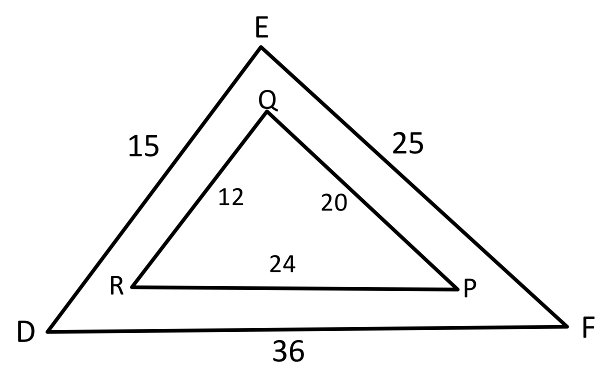
1. **Theorems and Postulates.**

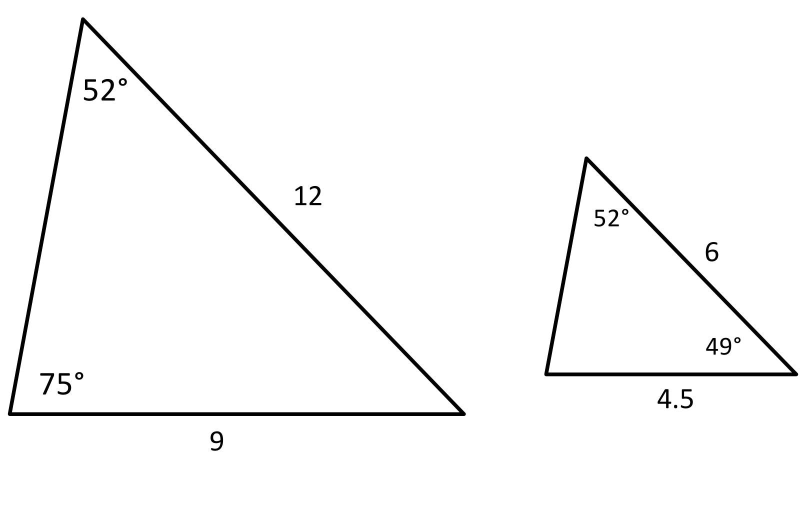
   a. State or illustrate the three triangle congruence postulates.
   b. State or draw the three triangle similarity postulates / theorems.

2. **Determining Similarity.** For the given polygons:

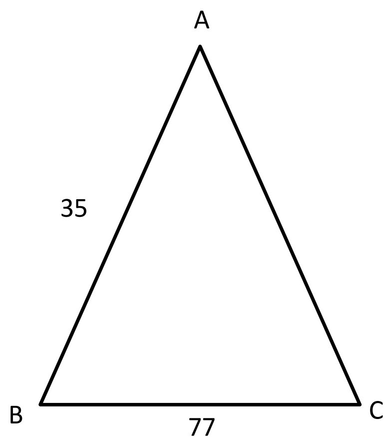
   - State whether they are similar.
   - If they are, state what postulate or theorem supports your conclusion.
   - If not, state why not.
   - Justify your conclusion by indicating corresponding angles and / showing proportionality of the sides.

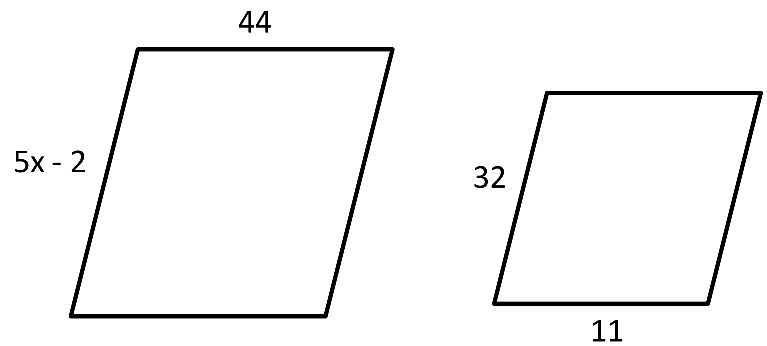




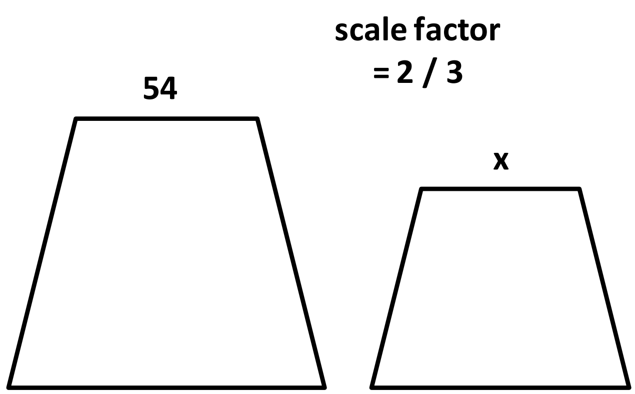


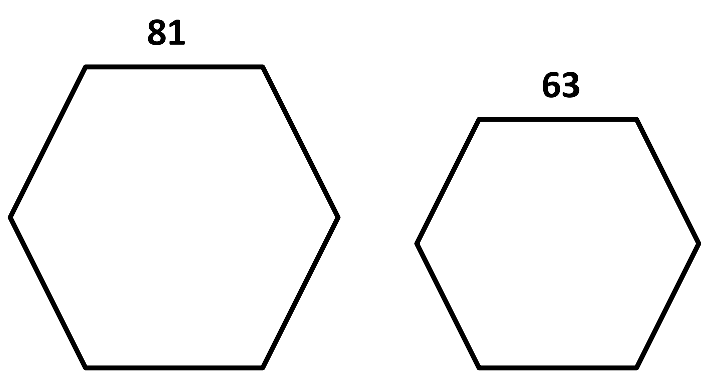
3. In the figures below, the polygons are similar. Find the value of *x*.





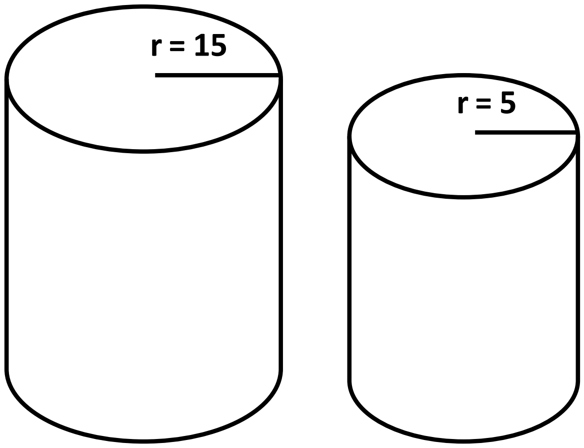
4. Scale Factors. Each of the following shapes is similar. Find either the scale factor or the indicated side length.

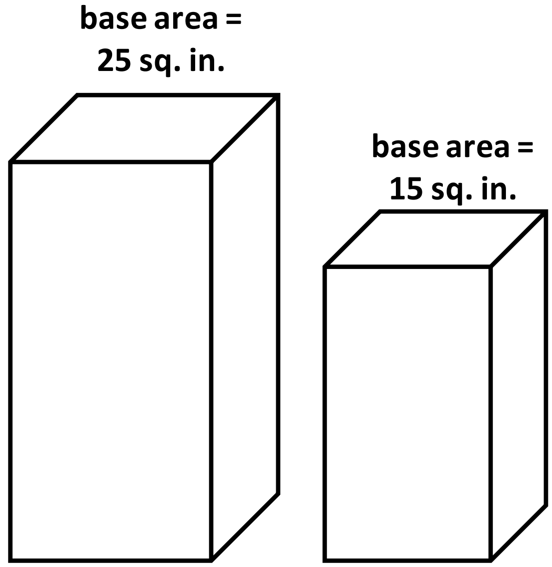




scale factor from large figure to small = x

5. **Area and Volume Ratios.** In the figures below, find the ratios of the base perimeters, base areas, and the volumes for each pair of similar shapes.





Name: ______________________________

Class: ______________________________

Date: ______________________________

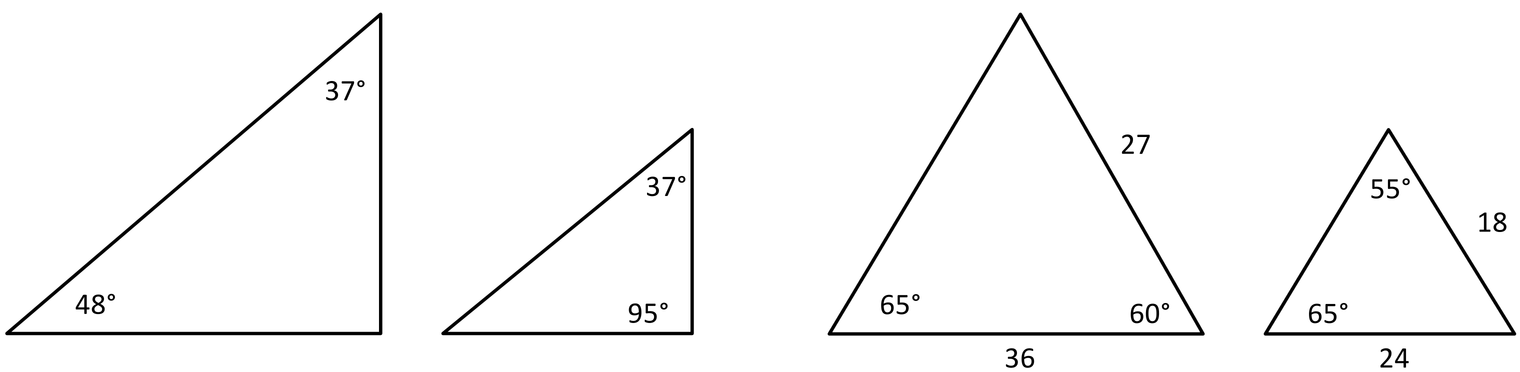
Congruence and Similarity Practice
Show work on graph paper and separate sheets of paper

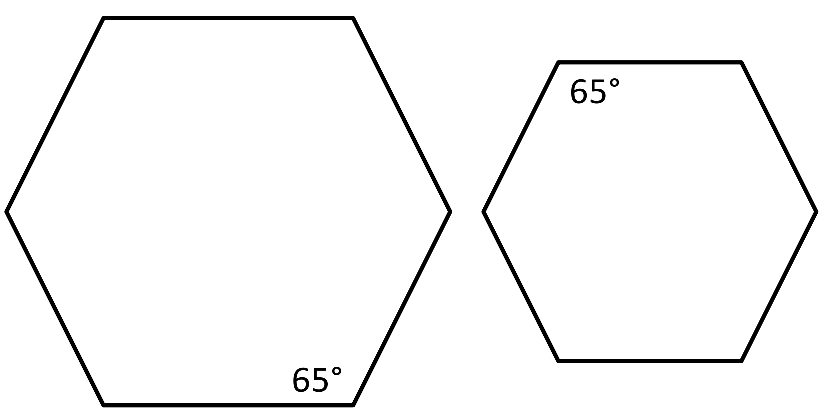
1. **Theorems and Postulates.**

   a. State the isosceles triangle theorem and its converse.
   b. State the H-L triangle congruence theorem.

2. **Determining Similarity.** For the given polygons:

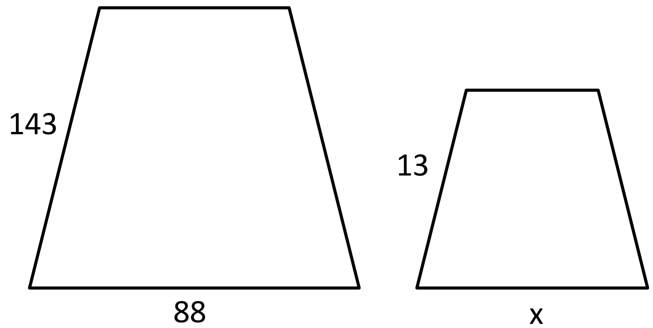
   - State whether they are similar.
   - If they are, state what postulate or theorem supports your conclusion.
   - If not, state why not.
   - Justify your conclusion by indicating corresponding angles and / showing proportionality of the sides.

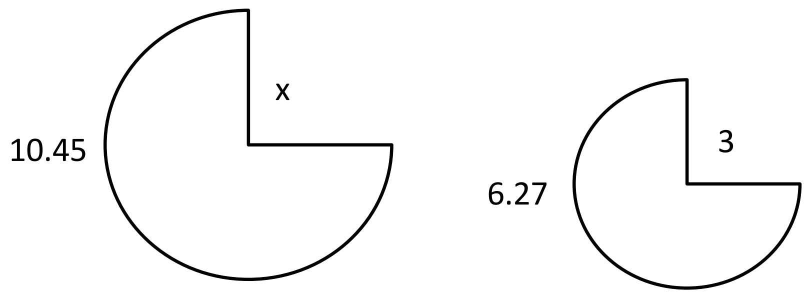




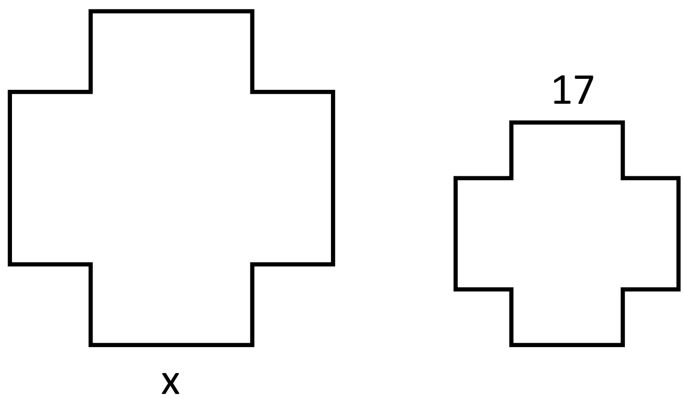
Both hexagons are regular

3. In the figures below, the shapes are similar. Find the value of *x*.

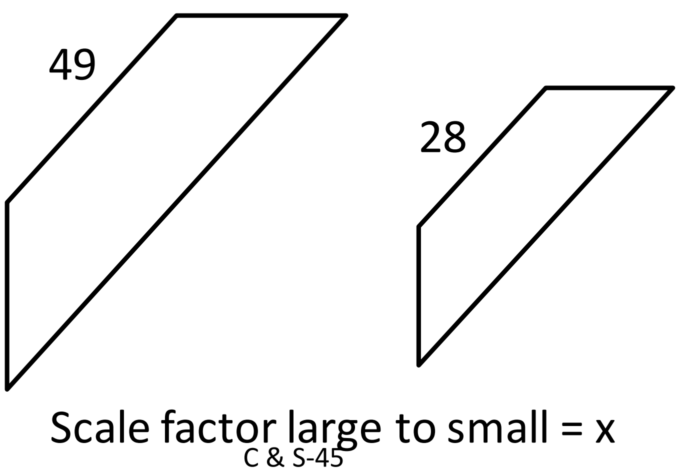




4. Scale Factors. Each of the following shapes is similar. Find either the scale factor or the indicated side length.

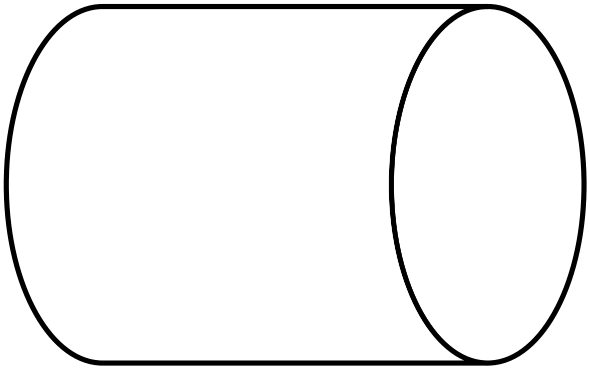


Scale factor = 5 / 4

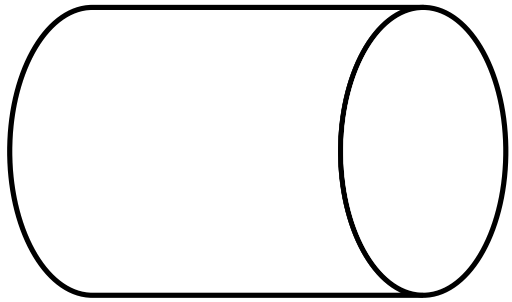


Scale factor large to small = x

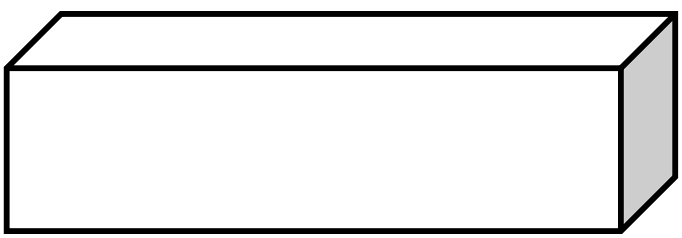
5. **Area and Volume Ratios.** In the figures below, find the ratios of the base perimeters, base areas, and the volumes for each pair of similar shapes.



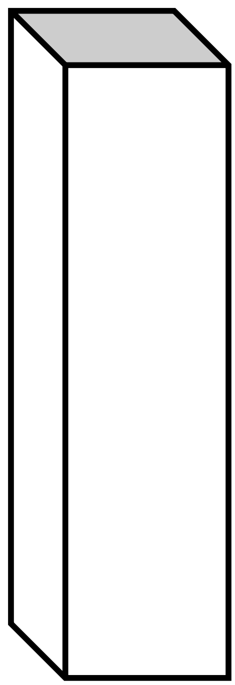
h = 35



h = 20

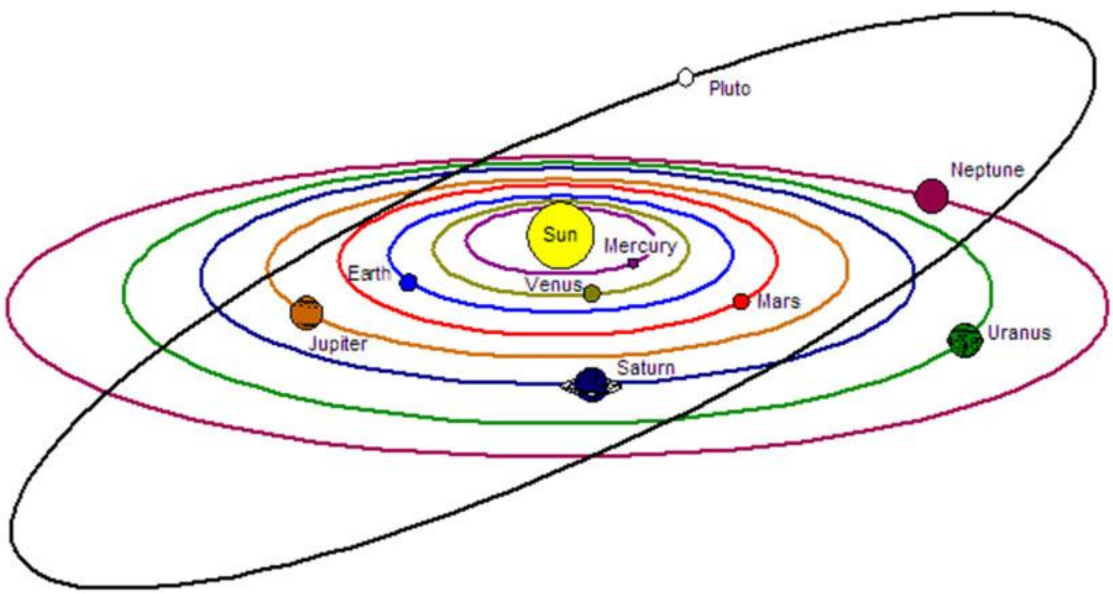


l = 20



h = 20

PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth



Circles

9th Grade Geometry

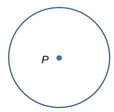
Captain Thomas R. Beall, U. S. Navy (Ret.)

Slide 1

Circles

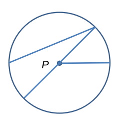
Definition

A circle is the set of all points in a plane that are of equal distance (equidistant) from a given point in a plane called the center of the circle.



Slide 2

Circles



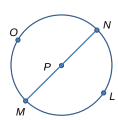
Radius (plural *radii*): a segment from the center of a circle to a point on the circle.

Chord: a segment whose endpoints lie on the circle.

Diameter: a chord that contains the center of a circle.

Slide 3

Circles



Arc: an unbroken part of a circle.

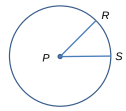
Semicircle: an arc whose endpoints are a diameter. A semicircle is named by its endpoints and another point that lies on the semicircle. $\overset{\frown}{MLN}$ is a semicircle.

Minor arc: an arc that is shorter than a semicircle of that circle. A minor arc is named by its endpoints. $\overset{\frown}{MO}$ is a minor arc.

Major arc: an arc that is longer than a semi-circle of that circle. . A major arc is named by its endpoints and another point that lies on the arc. $\overset{\frown}{MLO}$ is a major arc.

Slide 4

Circles

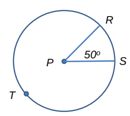


Central angle: an angle in the plane of a circle whose vertex is the center of the circle. <RPS is a central angle.

Intercepted arc: An arc whose endpoints lie on the sides of the angle and whose other points lie in the interior of the angle is the intercepted arc of the central angle.

Slide 5

Degree of Measure of Arcs



The degree of measure of a minor arc is the measure of its central angle. m$\overset{\frown}{RS}$ is 50°.

The degree of measure of a major arc is 360° minus the degree of measure of its minor arc. m$\overset{\frown}{STR}$ is 360° – 50° = 310°.

The degree of measure of a semicircle is 180°.

Slide 6

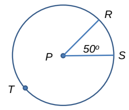
Arc Length

If *r* is the radius of a circle and *M* is the degree of measure of an arc of that circle, then the length *L* of the arc is given by:

$$L = \frac{M}{360}°(2\pi r)$$

Slide 9

Arc Measure and Length



The degree of measure of a minor arc is the measure of its central angle. $m\widehat{RS}$ is 50°.

The degree of measure of a major arc is 360° minus the degree of measure of its minor arc. $m\widehat{STR}$ is 360° – 50° = 310°.

The degree of measure of a semicircle is 180°.

If *r* is the radius of a circle and *M* is the degree of measure of an arc of that circle, then the length *L* of the arc is given by:

$$L = \frac{M}{360°}(2\pi r)$$

Slide 10

Homework Review

$$L = \frac{M}{(360°)}(2\pi r)$$

31. $m\angle P = 90°; r = 10$

$$L = \frac{90°}{360°}(2\pi 10) =$$

$$= \frac{1}{4}(2\pi 10)$$

$$= \frac{20}{4}\pi = 5\pi$$

35. $L = 20; r = 100$

$$20 = \frac{M}{360°}(2\pi 100)$$

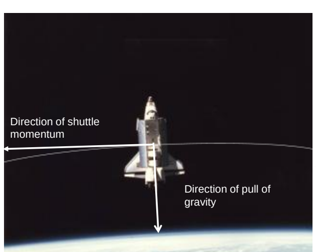
$$20 \times 360° = M(200\pi)$$

$$\frac{20}{200\pi} \times 360° = M$$

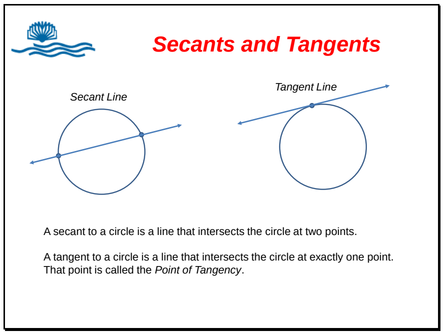
$$\frac{36°}{\pi} = M = 11.46°$$

Slide 11

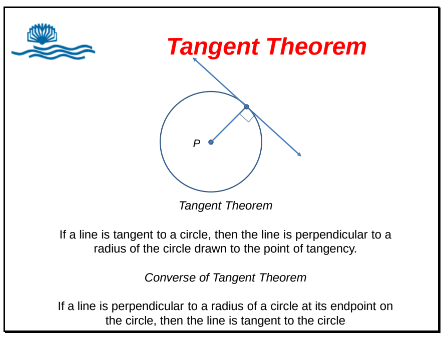
Tangents



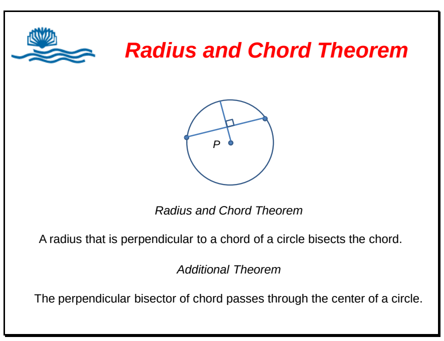
Slide 12



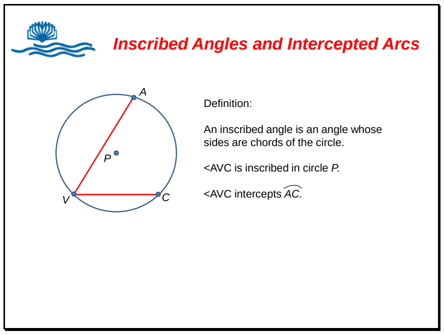
Slide 13



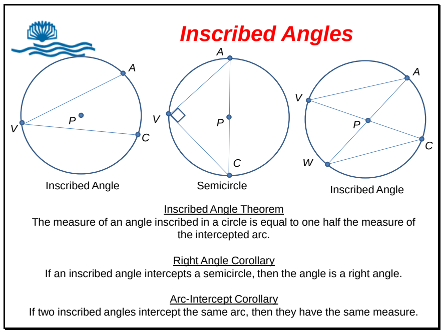
Slide 14



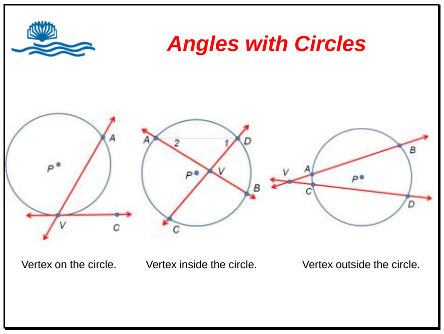
Slide 25



Slide 26



Slide 28

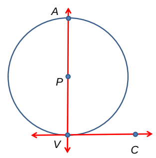


Slide 29

Angles Formed by Secants and Tangents

In this activity, you will examine three configurations of secant – tangent angles.

1. This secant-tangent angle is a right angle (note that the secant contains the center of the circle in this case.
   a. m∠AVC =
   b. Measure of arc *AV* =
   c. How does the relationship between the two measures compare with the one between an inscribed angle and an intercepted arc?

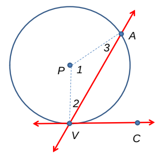


Slide 30

Angles Formed by Secants and Tangents

This secant – tangent angle is acute (less than 90°). Complete the following table:

m arc *AV*	m∠1	m∠2	m∠PVC	m∠AVC
120°	120°			
100°				
80°				
x°				



Complete the following statement:

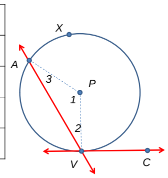
The measure of an acute secant-tangent angle to its vertex circle is the measure of its intercepted arc.

Slide 31

Angles Formed by Secants and Tangents

This secant – tangent angle is obtuse (greater than 90°). Complete the following table:

m arc *AXV*	m∠1	m∠2	m∠PVC	m∠AVC
200°	160°	10°	90°	100°
220°				
240°				
x°				



Complete the following statement:

The measure of an obtuse secant-tangent angle to its vertex circle is the measure of its intercepted arc.

Slide 32

Theorem

Based on your findings, complete the following theorem:

If a tangent and a secant (or chord) intersect on a circle at the point of tangency, then the measure of the angle formed is _______ the measure of its intercepted arc.

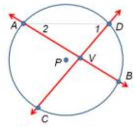
Slide 33

Vertex Inside the Circle

∠AVC is an exterior angle of Δ*ADV*. What is the relationship between the measure of *∠AVC* and the measures of *∠1* and *∠2*?

Complete the following table:

m arc *AC*	m arc *BD*	m∠1	m∠2	m∠AVC	m∠DVB
160°	40°	80°	20°	100°	100°
180°	70°	90	35	125	125
220°	60°				
x°					



Based on your results, complete the following theorem:

The measure of an angle formed by two secants or chords that intersect in the interior of a circle is ____________ the ____________ of the measures of the arcs intercepted by its angle and its vertical angle.

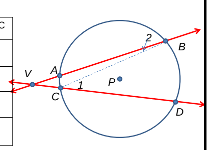
Slide 34

Vertex Outside the Circle

∠1 is an exterior angle of Δ*BVC*. What is the relationship between the measure of *∠1* and the measures of *∠2* and *∠AVC*?

Complete the following table:

m arc *BD*	m arc *AC*	m∠1	m∠2	m∠AVC
200°	40°	100°	20°	80°
250°	60°			
100°	50°			
x°				



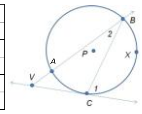
Based on your results, complete the following theorem:

The measure of an angle formed by two secants that intersect in the exterior of a circle is one half the difference of the measures of the intercepted arcs.

Slide 35

Vertices Outside the Circle

m arc BXC	m arc AC	m ∠1	m ∠2	m ∠AVC
250°	60°	125°	30°	95°
200°	40°	100°	20°	80°
130°	40°	65°	20°	45°
70°	30°	35°	15°	20°
$x_1°$	$x_2°$	$0.5x_1°$	$0.5x_2°$	$0.5(x_1° - x_2°)$



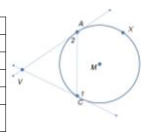
Theorem: The measure of the secant - tangent angle with its vertex outside the circle is one – half the difference of the measures of the intercepted arcs.

Theorem: The measure of a tangent – tangent angle with its vertex outside the circle is one half the difference of the measures of the intercepted arcs or the measure of the major arc minus 180 .

Slide 36

Vertices Outside the Circle

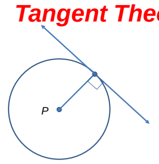
m arc AXC	m arc AC	m ∠1	m ∠2	m ∠AVC
300°	60°	150°	30°	120°
250°	110°	125°	55°	70°
220°	140°	110°	70°	40°
200°	160°	100°	80°	20°
$x_1°$	$360° - x_1°$	$0.5x_1°$	$0.5(360° - x_1°)$	$180° - x_1°$



Theorem: The measure of a tangent – tangent angle with its vertex outside the circle is one half the difference of the measures of the intercepted arcs or the measure of the major arc minus 180 .

Slide 37

Tangent Theorem



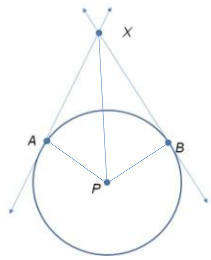
Tangent Theorem

If a line is tangent to a circle, then the line is perpendicular to a radius of the circle drawn to the point of tangency.

Converse of Tangent Theorem

If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle

Slide 38



Paragraph Proof

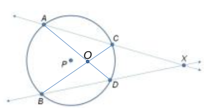
1.I know that AP = BP because they are radii of the same circle.

2.I know that PX = PX by the reflexive property.

3.I know that m∠XBP = m∠XAP = 90° because if a line is tangent to a circle then the line is perpendicular ot a radius of the circle drawn to the point of tangency.

4.Therefore, I can show that AX = BX by the Pythagorean Theorem since the measures of the two other corresponding sides of the right triangles are equal.

Slide 39



1. ΔADX and ΔBCX
2. I know that m∠A = m ∠B because they are inscribed and intercept the same arc.
3. I also know that m∠X = m ∠X by the reflexive property.
4. Therefore I know that ΔADX and ΔBCX are similar by the AA similarity property.
5. Because they are similar, we know that:

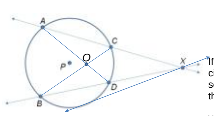
$$\frac{AX}{BX} = \frac{XD}{XC} \Rightarrow AX * XC = BX * XD$$

Theorem

If two secants intersect outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lenghts of the other secant segment and its external segment:

Whole * Outside = Whole * Outside

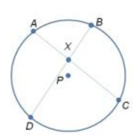
Slide 40



Theorem

If the secant and tangent intersect outside the circle, then the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared.

Whole * Outside = Tangent Squared



Theorem

If two chords intersect inside a circle, then the product of the lengths of the segments of one chord equals the product of the lengths of the segments of the other chord.

Name: ________________________

Class:_________________________

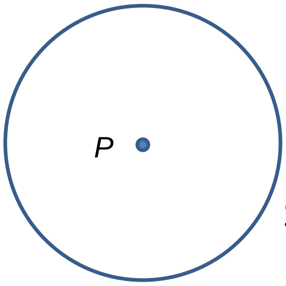
Date:_________________________

Notetaking Sheet – Circle Definitions

1. A circle is: ____

____

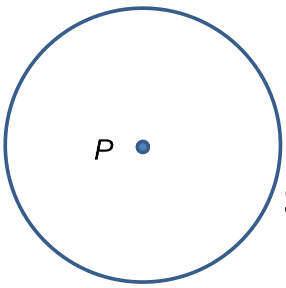
____



2. A radius is: ____

____

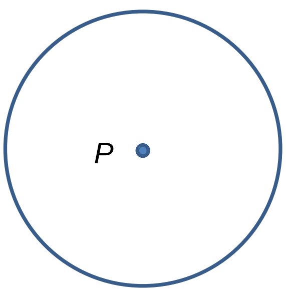
____



3. A chord is: ____

____

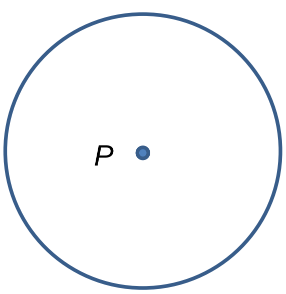
____



4. A diameter is: ____

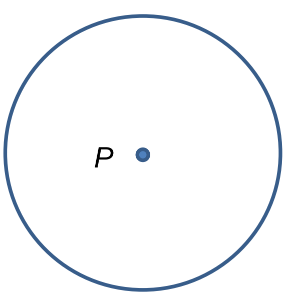
____

____



5. Central angle: an angle in the plane of a circle whose ________________

____



6. Intercepted arc: An arc whose endpoints lie on the sides of the angle ___

____

7. The degree of measure of a minor arc is ______________________________

____

____

8. The degree of measure of a major arc is ______________________________

____

____

9. The degree of measure of a semicircle is ______________________________

10. Arc Length: ____

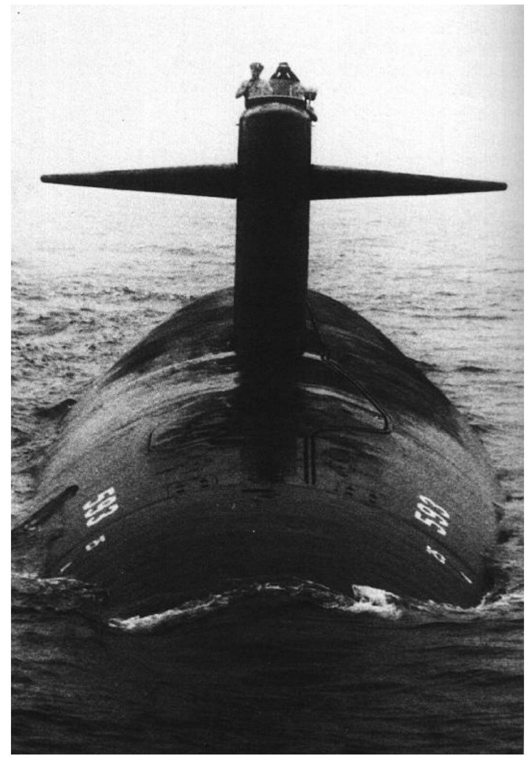
____

____

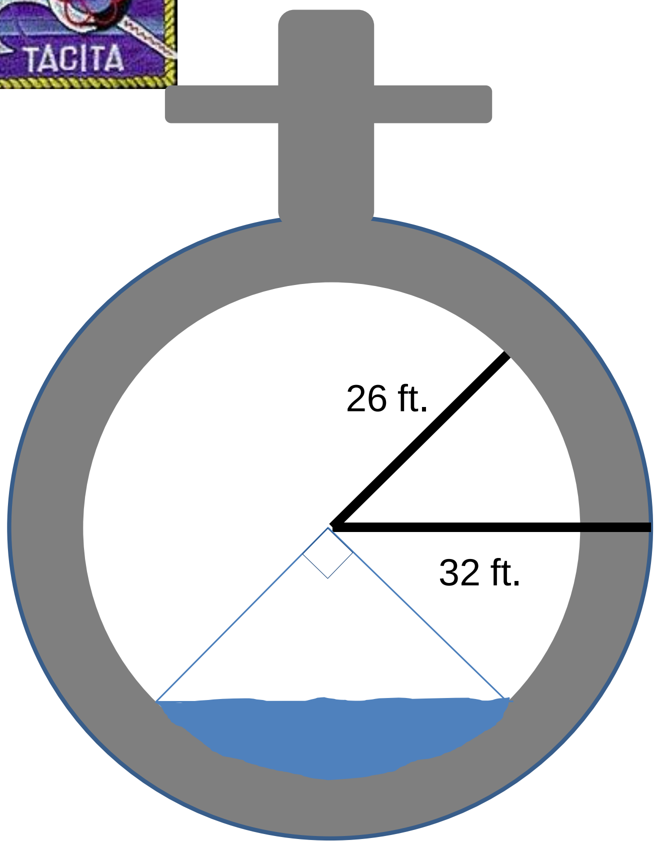
$L =$ ______________________________

Loss of U. S. S. Thresher

April 10th, 1963



Loss of U. S. S. Thresher



1. If the reactor / engine compartment is 100 ft. long, what is the volume of the flooded portion?

2. How many gallons of water flooded into the compartment? (7.48 gallons per cu. ft.)

3. What was the added weight to the submarine? (8.35 gallons to one pound)

Name: ______________________

Class: ______________________

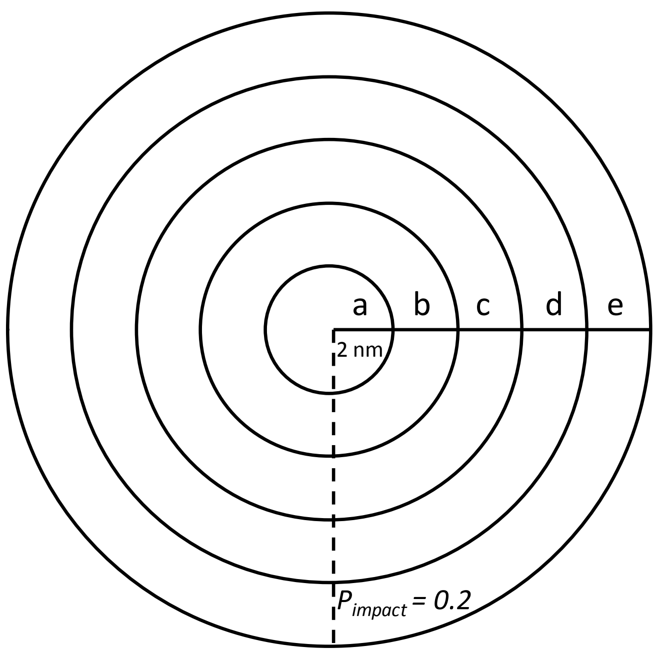
Date: ______________________

Practical Problems Involving Circles
Solve All Problems on Separate Sheets of Paper

1. A test missile is fired from the Pacific Missile Test Center at Edwards Air Force Base in California to an impact area in the Pacific Ocean. The impact area depicted below is a circle 20 nautical miles (nm) in diameter.

   - There are five circular regions in the impact area, labeled *a, b, c, d, e*. Each is 2 (nm) in width.

   - The probability that the missile will impact in any one of the five circular regions (P_{impact}) is 20% (0.20). Therefore, the probability that the missile will impact somewhere in the impact area is P_{impact}(a) + P_{impact}(b) + P_{impact}(c) + P_{impact}(d) + P_{impact}(e) $= 0.20 + 0.20 + 0.20 + 0.20 + 0.20 = 1.00 = 100\%$.

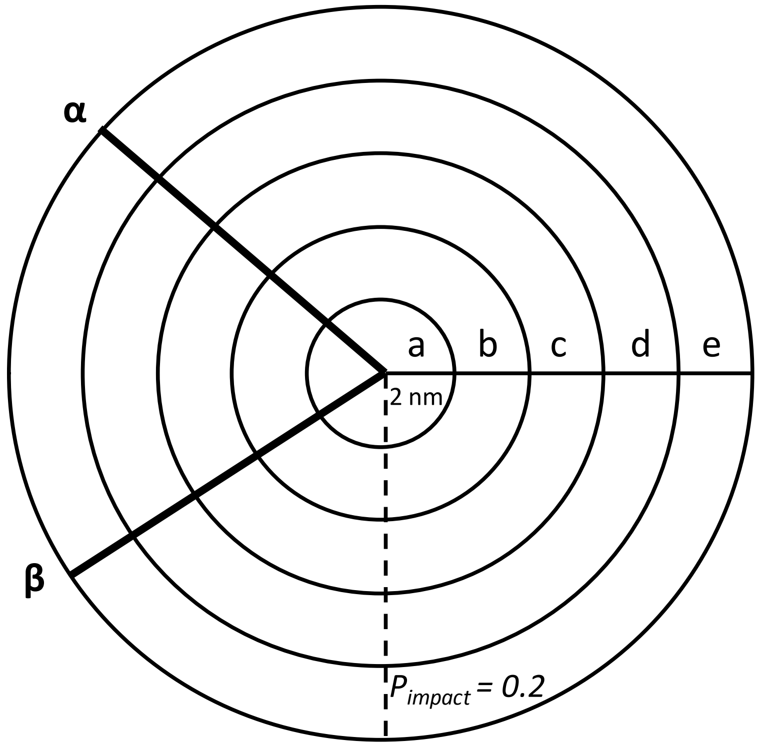


   a. What is the total impact area?

   b. What is the area of region c?

   c. What is the probability that the missile will impact no closer than 6 nm to the center of the impact area?

2. The measure of the central angle intercepting the minor arc αβ is 85°.



a. What is the length of minor arc αβ?

b. What is the probability that the missile will impact in the region between the two radii that intercept minor arc αβ?

c. What is the probability that the missile will impact in the region between the two radii that intercept minor arc αβ no closer than 6 nm from the center?

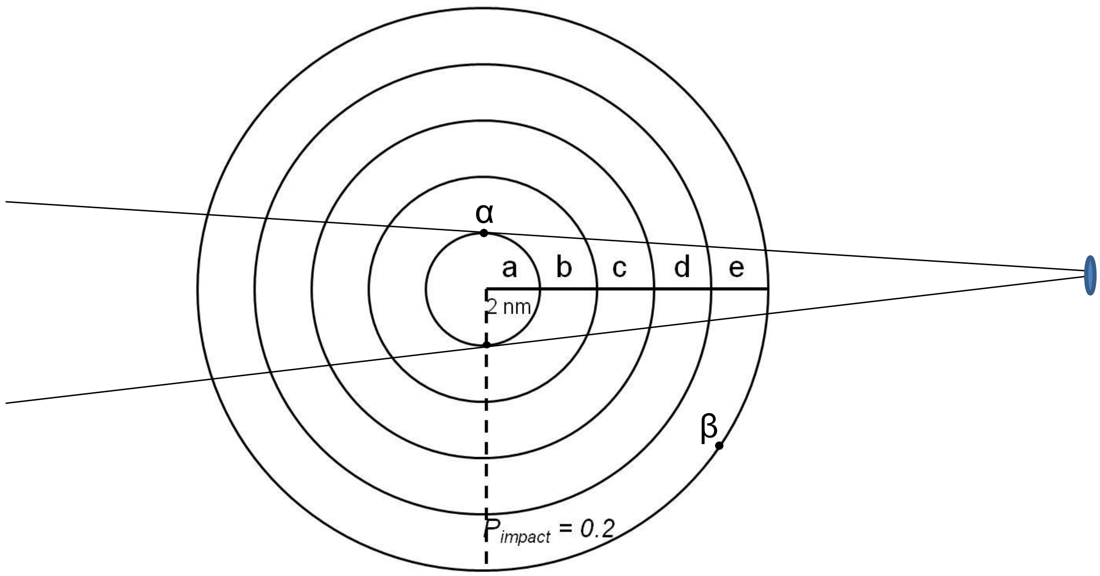
Name: ________________________

Class:_________________________

Date:_________________________

Practical Problems Involving Circles
Solve All Problems on Separate Sheets of Paper

A Chinese surveillance ship is sitting outside of the impact area using radar to observe the missile's impact.



1. If the measures of the two intercepted arcs on the outer edge of region *e* are 25° and 20° respectively, what is the measure of the angle formed by the radar beam?

2. If the radar beam covers one fifth of the impact area every second, what is the probability that the Chinese surveillance ship will see the missile just as it hits the ocean?

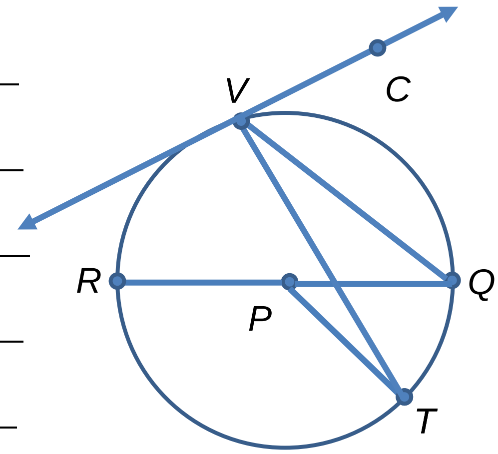
3. If the length of the radar beam from the Chinese surveillance ship to point α is 20 nm, what is the distance of the Chinese surveillance ship from the center of the impact area?

4. If the Chinese surveillance ship were to move from its present position to the edge of the impact area at point β and shine its radar beam toward the center of the impact area, what would be the measure and arc length of the arc intercepted on the outer edge of region *e*?

Name: ______________________________

Geometry Review I - Circles

1. **Vocabulary**. Identify what the given segment, angle, or arc is (i.e. radius, chord, etc.):



   a. Segment VQ: ______________________________

   b. Segment PQ: ______________________________

   c. Angle <QPT: ______________________________

   d. Angle <QVT: ______________________________

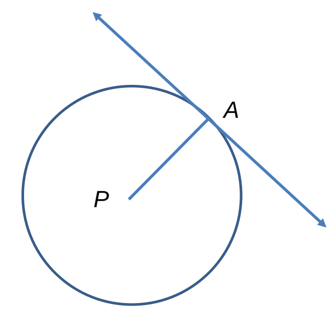
   e. Segment RQ: ______________________________

   f. Line VC: ______________________________

   g. Point V: ______________________________

   h. Angle <QVC: ______________________________

2. **Theorems**. We have studied many theorems about the relationships between circles and lines, segments, and angles. Given the pictures below, answer the questions about these relationships.

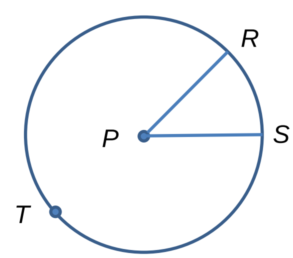


   a. The angle formed between radius PA and the line A is

   _______________ degrees.

   b. The measure of ∠RPS is equal to the measure of

   ______________________________.

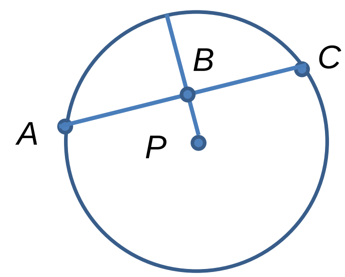


   c. If radius PR is 5 and measure of arc RS is 20°, then what

   formula gives the length of arc RS?

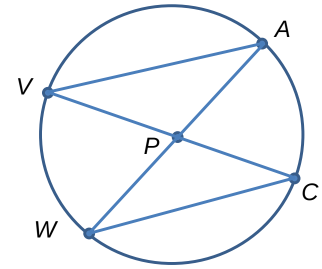
   ____

d. Chord AC is perpendicular to the radius. What is the relationship between segments AB and AC?



____

e. Arc AC = 60°, arc VA = 120°. Find the measures of the following angles:

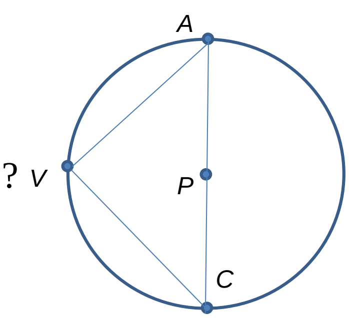


i. ∠AVC: ______________________________________

ii. ∠APV: ______________________________________

iii. ∠CPW: ______________________________________

iv. ∠WPV: ______________________________________

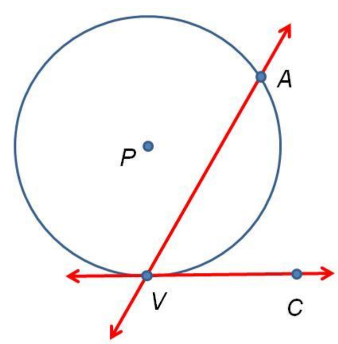


f. If segment AC is a diameter, what is the measure of ∠AVC? What is the measure of arc AC?

____

g. If arc AV = 120°, what is the measure of ∠AVC?

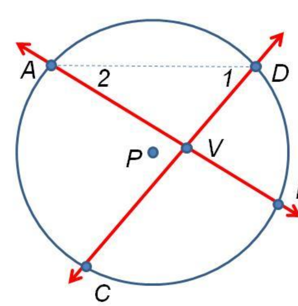
____



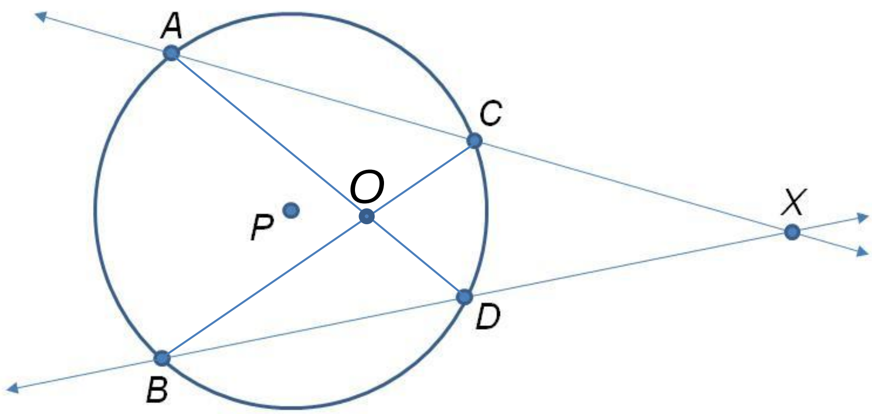
h. Point V is called the ______________________________

i. The measure of arc AC = 110° and the measure of arc DB = 70°. What is the measure of ∠DVB? __________________

What is the measure of ∠BVC? ______________________



3. For the following questions, refer to the illustration below.



a. The m arc AB = 60° and m arc CD = 35°. What is the m∠X?

____

b. The m ∠AOC = 100°. What is m∠AOB?

____

c. The m arc AB = 60° and m arc CD = 35°. What are m∠CAD and m∠ACB?

____

Name: ______________________________

Date: ______________________________

Class: ______________________________

Geometry Review II: Circles

Show all work on separate sheets of paper.

8. Determine the length of an arc with given central angle measure, $m\angle P$, in a circle with radius *r*. Round your answers to the nearest hundreth.

    a. $m\angle P = 55°; r = 15$ b. $m\angle P = 144°; r = 17$ c. $m\angle P = 27°; r = 36$

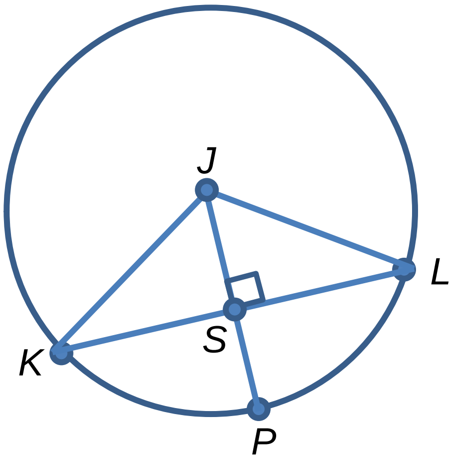
9. Determine the degree measure of an arc with given length, *L*, in a circle with radius *r*. Round your answers to the nearest hundredth.

    a. $L = 75, r = 15$ b. $L = 27, r = 15$ c. $L = 100, r = 79$

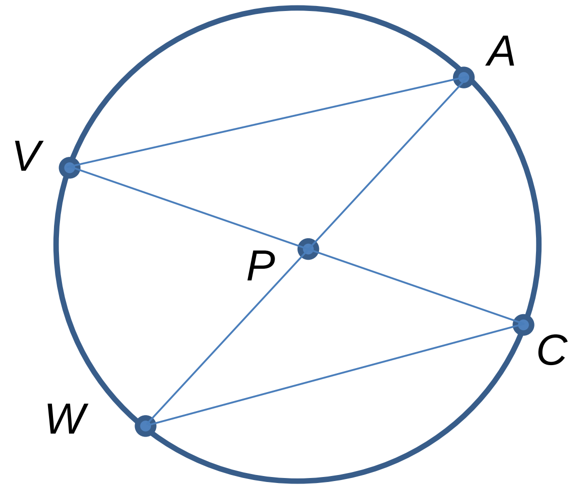
10. Refer to circle *J*, in which *JP* is perpendicular to *KL* at *S*. Round your answers to the nearest hundredth.

    a. If *JL* = 20 and *JS* = 6, what is *KS*? What is *SL*?

    b. If *JK* = 60 and *JS* = 30, what is *KS*? What is *SL*?



11. In circle P, $m\angle APC = 70°$. *WA* and *VC* are diameters. Find the following measures:



a. The measure of arc *AC* _______________

b. $m\angle AVC$: ___________________________

c. $m\angle AWC$: ___________________________

d. $m\angle APV$: ___________________________

e. The measure of arc *VA*: ______________

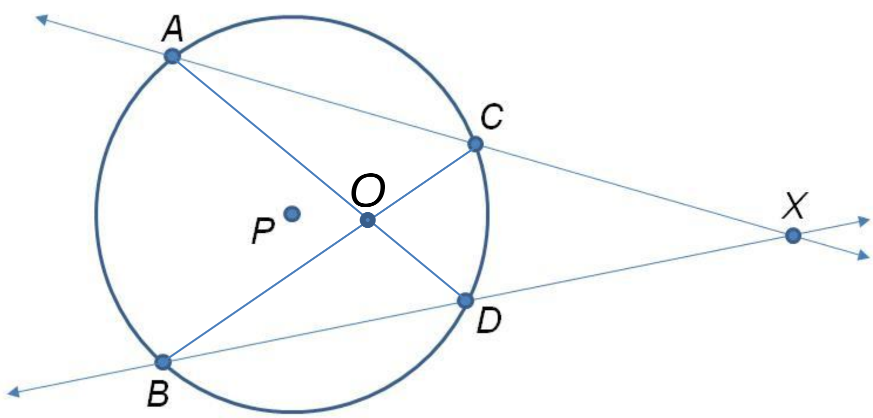
f. $m\angle VPW$: ___________________________

g. The measure of arc *VW*: ______________

h. $m\angle VAW$: ___________________________

i. The measure or arc *VCW*: ______________

3. For the following questions, refer to the illustration below.



a. Segment AX = 11, segment CX = 6, and segment DX = 8. Find the length of segment BX.

____

b. Segment OA = 3, segment OB = 3.5, segment OC = 2. Find the length of segment OD.

____

c. Arc AB = 100°, arc CD = 50°. Find the measure of ∠AXB.

____

Name: ___________________________________

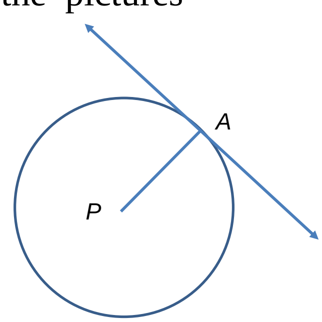
Geometry Assessment – Circles

1. **Vocabulary.** Write the number of the definition on the right next to its term on the left:

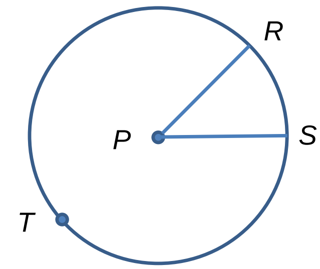
a. Chord. ____________ b. Tangent Line. ____________ c. Secant. ____________ d. Radius. ____________ e. Inscribed Angle. ____________ f. Central Angle. ____________ g. Intercepted Arc. ____________	1. An angle whose vertex lies on the circle. 2. A line segment with one endpoint at the center of the circle and the other on the circle. 3. A line segment whose endpoints lie on a circle. 4. An arc of a circle whose endpoints intersect the rays of an angle. 5. A line passing through exactly one point on a circle. 6. An angle whose vertex is the center point of a circle. 7. A line passing through two points on a circle.

2. **Theorems**. We have studied many theorems about the relationships between circles and lines, segments, and angles. Given the pictures below, answer the questions about these relationships.

   a. Segment PA is ____________________________ to line A.



b. If m∠RPS = 45°, m arc **RTS** = ________________________ .

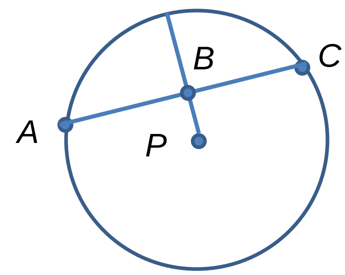


c. If radius PR is 8 and measure of arc RS is 30°, then what is the length of arc RS?

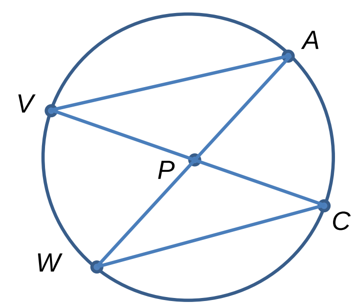
____

d. Segment AB = Segment AC. What is the measure of ∠ABP?

____



e. The measure of arc AC = the measure of arc VW = 75°. Segments VC and AW are diameters. Find the measures of the following angles:



i. ∠AVC: _____________________________________

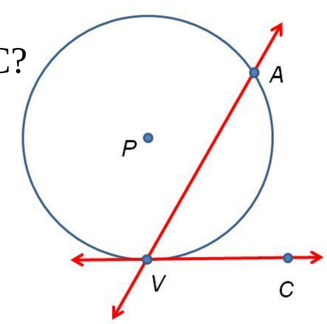
ii. ∠APV: _____________________________________

iii. ∠CPW: _____________________________________

iv. ∠WPV: _____________________________________

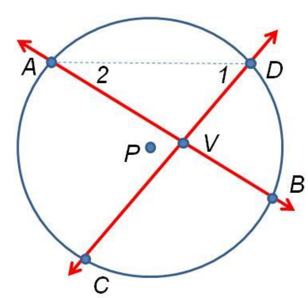
f. The measure of arc AV = 140°. What is the measure of ∠AVC?

____



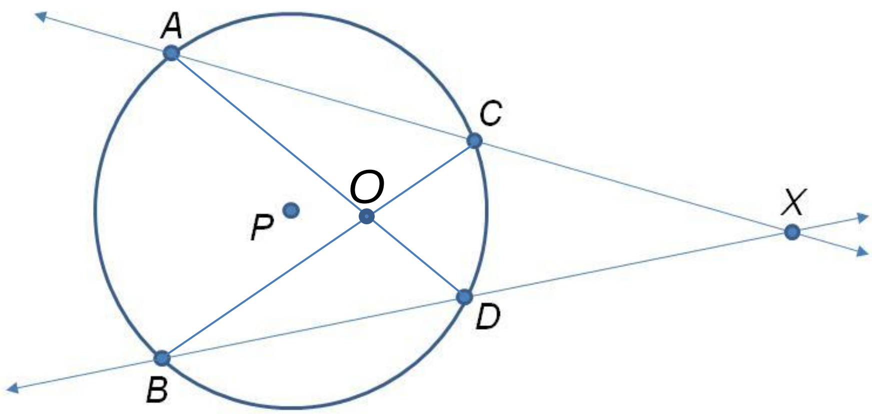
g. If I draw a segment from point P to point V, what is the measure of ∠PVA? __________________________________

h. The measure of arc AC = 135° and the measure of arc DB = 55°. What is the measure of ∠DVB? ___________________



What is the measure of ∠BVC? _______________________

3. For the following questions, refer to the illustration below.



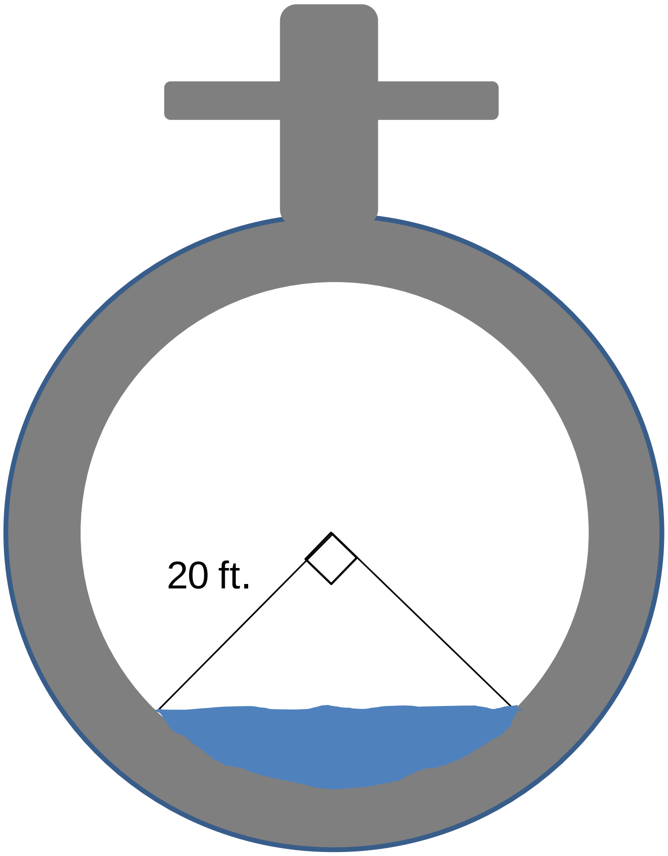
a. Segment AX = 10, segment CX = 5, and segment DX = 6.25. Find the length of segment BX.

____

b. The measure of arc AB = 115°, and the measure of arc CD = 70°. Find the measure of ∠AXB.

____

4. The submarine U. S. S. *Squalus* sank on May 23rd, 1939 when her aft torpedo compartment flooded. Answer the following questions:



a. If the aft torpedo compartment is 60 ft. long, what is the volume of the flooded portion?

____

b. How many gallons of water flooded into the compartment? (7.48 gallons per cu. ft.)

____

c. What was the added weight to the submarine? (8.35 gallons to one pound)

____

Name: ______________________________

Date: _______________________________

Advisor: _______________________________

Project: Constructing a Scale Model of the Solar System
(Answer questions on these sheets)

Suppose you are building a scale model of the solar system. Use the table below to determine the model sizes of the planets and the model distances of the planets from the sun.

	Planet Diameter	Planet Distance from Sun
Sun	1,391,900 km	
Mercury	4866 km	57,950,000 km
Venus	12,106 km	108,110,000 km
Earth	12,742 km	149,570,000 km
Mars	6760 km	227,840,000 km
Jupiter	139,516 km	778,140,000 km
Saturn	116,438 km	1,427,000,000 km
Uranus	46,940 km	2,870,300,000 km
Neptune	45,432 km	4,499,900,000 km
Pluto	3400 km	5,913,000,000 km

1. Select a convenient size to make the model of the sun. What are the scale factors of your model? (Note: You may use different scale factors for model diameter and model distance.)

2. Using your scale factors from step 1, find the sizes of the planets in your model and model the distances from the Sun.

	Model Planet Diameter	Model Planet Distance from Sun
Sun		
Mercury		
Venus		
Earth		
Mars		
Jupiter		
Saturn		
Uranus		
Neptune		
Pluto		

3. What is the ratio of the volume of the sun to the volume of your model of the sun?

4. What problems would you have in making your model? How big a space would you need for it? Would the smaller planets be big enough for you to see them well?

Proposal for Model of the Solar System

Neatly handwritten or typed on separate sheets of paper attached to this one (the neater it is, the more credit you get)

1. **Briefly state what it is you want your model to portray.** The model must incorporate

   a. Scaled distances of the planets from the Sun.
   b. Scaled sizes of the Sun and the planets.
   c. Information about each planet.

2. **Provide a description of what your model will look like and where it will be located.** Include answers to the following questions:

   a. Will it be two dimensional or three dimensional?
   b. Will it be located inside or outside (if outside, where)?

3. **Describe in detail what materials you will use to construct your model and where you will set it up.**

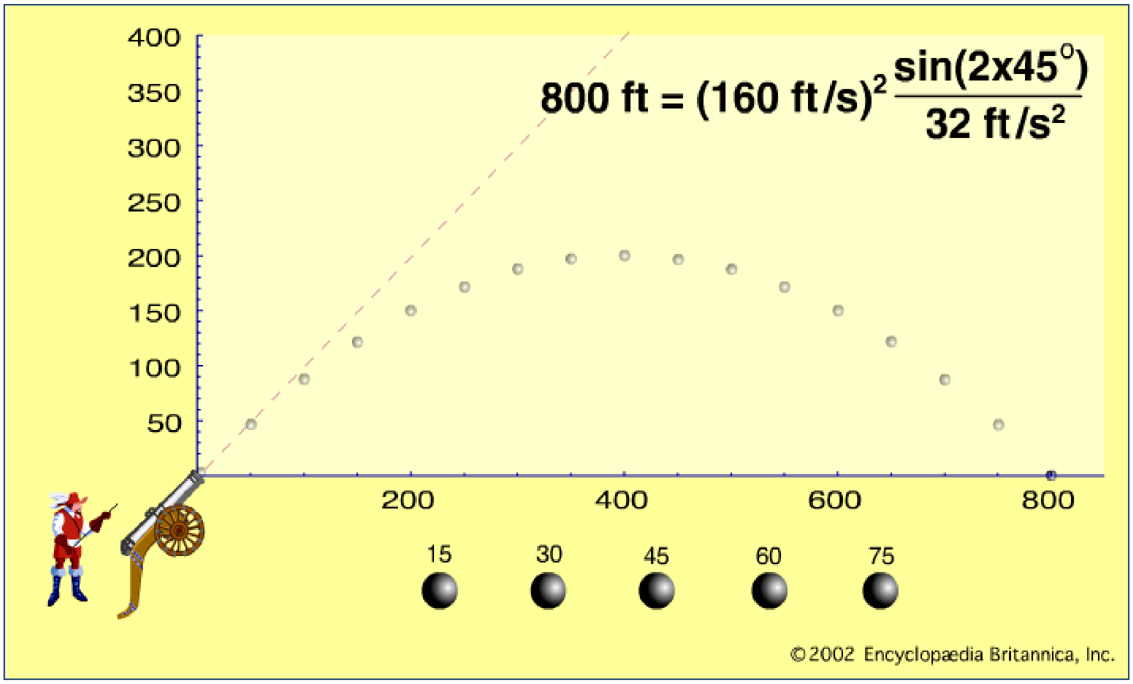
4. **Draw a picture of what you envision your model will look like.**

Feel free to use online resources to guide you. Some of these include:

http://www.exploratorium.edu/ronh/solar_system/

http://www.educationbug.org/a/make-a-model-solar-system-science-project.html

PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth



Trigonometry

9th Grade Geometry

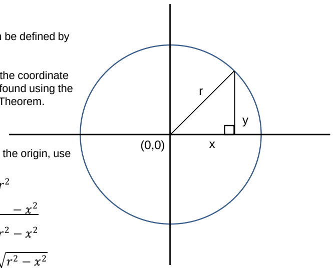
Captain Thomas R. Beall, U. S. Navy (Ret.)

Slide 1

Circles in the Coordinate Plane

1. Any circle can be defined by its radius.
2. Any radius in the coordinate plane can be found using the Pythagorean Theorem.
3. For a circle at the origin, use the equation:

$$x^2 + y^2 = r^2$$
$$-x^2 \qquad -x^2$$
$$y^2 = r^2 - x^2$$
$$y = \sqrt{r^2 - x^2}$$

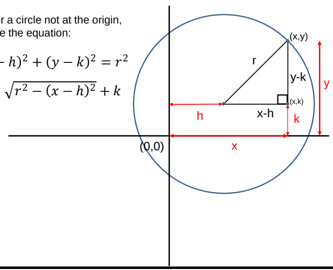


Slide 2

Circles in the Coordinate Plane

4. For a circle not at the origin, use the equation:

$$(x-h)^2 + (y-k)^2 = r^2$$
$$y = \sqrt{r^2 - (x-h)^2} + k$$



Slide 3

Graphing a Circle

1. Given:

   $x^2 + y^2 = 25^2$

2. Find the intercepts and plot them:

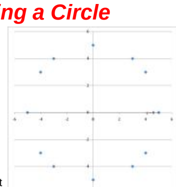
   $x^2 = 25$

   $x = \pm 5$

   $y^2 = 25$

   $y = \pm 5$

3. Set x and y to some other values and plot them:

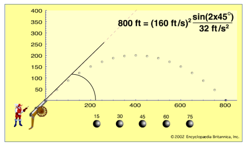


x	y	Points on Graph
3	$\pm$ 4	(3,4), (3,-4)
-3	$\pm$ 4	(-3,4), (-3,-4)
4	$\pm$ 3	(4,3), (4,-3)
-4	$\pm$ 3	(-4,3), (-4,-3)

Slide 4

Trigonometry

Trigonometry developed from a need to compute angles and distances in such fields as astronomy, map making, surveying, and artillery range finding. Problems involving angles and distances in one plane are covered in **plane trigonometry**. Applications to similar problems in more than one plane of three-dimensional space are considered in **spherical trigonometry**.



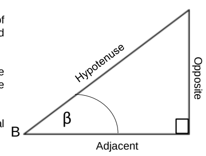
Slide 5

Basics

Trigonometry is simply the study of angles and measures in triangles and the relationships between them.

Given an angle measure and one side measure or two side measures, we can usually find the other measures.

Can you think of some practical applications of this?

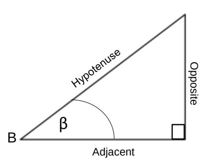


Slide 6

Tangent Ratio

For a given ∠B with measure of β°, the tangent of ∠B or tan β, is the ratio of the leg opposite ∠B to the length of the length adjacent to ∠B in any right triangle having ∠B as one vertex or:

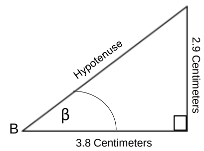
$$\tan\beta = \frac{opposite}{adjacent}$$



Slide 7

Example

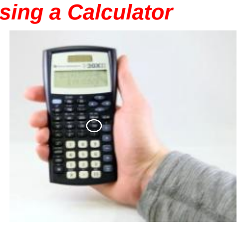
$$tan\ \beta = \frac{2.9}{3.8} = 0.76$$



Slide 8

Using a Calculator

β	tanβ
15°	
30°	
45°	
60°	
75°	



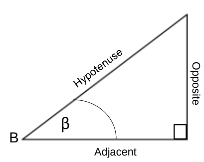
Function and Key	Given (input)	What to Find (output)
Tangent $\boxed{\text{tan}}$	angle measure	tangent ratio
Inverse tangent $\boxed{\text{tan}^{-1}}$	tangent ratio	angle measure

Slide 9

Sine and Cosine Ratios

For a given ∠B with measure of β°, the sine of ∠B or sin β, is the ratio of the leg opposite ∠B to the length of the hypotenuse in a right triangle with ∠B in as one vertex or:

$$\sin\beta = \frac{opposite}{hypotenuse}$$

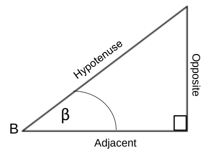


Slide 10

Sine and Cosine Ratios

For a given $\angle B$ with measure of $\beta°$, the cosine of $\angle B$ or $\cos \beta$, is the ratio of the leg adjacent $\angle B$ to the length of the hypotenuse in a right triangle with $\angle B$ in as one vertex or:

$$\cos \beta = \frac{adjacent}{hypotenuse}$$



Slide 11

Relationship Between Sine, Cosine, and Tangent

$$\frac{\sin \beta}{\cos \beta} = \frac{\frac{opposite}{hypotenuse}}{\frac{adjacent}{hypotenuse}} = \frac{opposite}{adjacent} = \tan \beta$$

Slide 12

Another Relationship

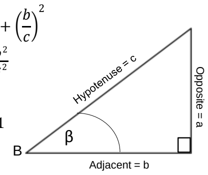
β	$\sin \beta$	$\cos \beta$	$(\sin \beta)^2 + (\cos \beta)^2$
20°			
40°			
60°			

$$(\sin \beta)^2 + (\cos \beta)^2 = 1$$

Slide 13

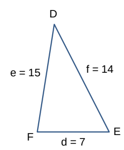
Proof

$$(\sin\beta)^2 + (\cos\beta)^2 = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$
$$= \frac{a^2}{c^2} + \frac{b^2}{c^2}$$
$$= \frac{a^2+b^2}{c^2}$$
$$= \frac{c^2}{c^2} = 1$$



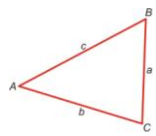
Slide 14

Solving Triangles Using Law of Sines & Law of Cosines



Slide 15

Law of Sines



For ***any triangle*** ΔABC with sides *a, b,* and *c*:

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

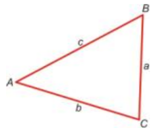
Law of Sines can be used when:

1. Measures of two angles and a side (AAS and ASA) or,
2. Measures of two sides and a ***non-included*** angle are known.

Cannot be used when the measures of two sides and an included angle (SAS) or three sides (SSS) are known. ***Why not?***

Slide 16

Solving Triangles Using Law of Sines

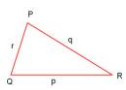


$m\angle A = 56°, m\angle B = 24°, b = 1.22\ cm$

1. $m\angle C = 180° - 56° - 24° = 100°$
2. $\frac{\sin B}{b} = \frac{\sin 24°}{b} \cong \frac{0.41}{1.22} \cong 0.34$
3. $\frac{\sin C}{c} = \frac{\sin 100°}{c}$
4. $c = \frac{\sin 100°}{0.34} \cong 2.87$

Slide 17

Solving Triangles Using Law of Sines



$m\angle P = 30°, m\angle Q = 40°, q = 10$

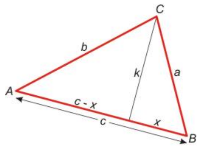
1. Find $m\angle R \Rightarrow 180° - 30° - 40° = 110°$
2. $\frac{\sin P}{p} = \frac{\sin q}{q} = \frac{\sin r}{r} \Rightarrow \frac{\sin 30°}{p} = \frac{\sin 40°}{10} = \frac{\sin 110°}{r}$
3. Solve for p: $\frac{\sin 30°}{p} = \frac{\sin 40°}{10} \Rightarrow 10 \sin 30° = p \sin 40°$

   $(10)(0.5) = (p)(0.64) \Rightarrow p = \frac{(10)(0.5)}{0.64} = 7.81$
4. Solve for r: $\frac{\sin 30°}{7.81} = \frac{\sin 110°}{r} \Rightarrow 7.81 \sin 110° = r \sin 30°$

   $\frac{(7.81)(0.94)}{0.5} = 14.68$

Slide 18

Law of Cosines



$b^2 = a^2 + c^2 - 2ac \cos B$

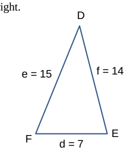
$a^2 = b^2 + c^2 - 2bc \cos A$

$c^2 = a^2 + b^2 - 2ab \cos C$

Slide 19

Solving Triangles Using Law of Sines & Law of Cosines

Find the measures of ∠D, ∠E, and ∠F in the triangle at right.



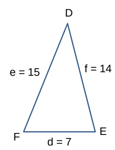
1. Use the Law of Cosines to find the first angle.

$e^2 = d^2 + f^2 - 2df\cos E$
$15^2 = 7^2 + 14^2 - 2(7)(14)\cos E$
$225 = 245 - 196\cos E$
$-20 = -196\cos E$
$\frac{-20}{-196} = \cos E$
$m\angle E = \cos^{-1}(\frac{-20}{-196}) = 84°$

Slide 20

Solving Triangles Using Law of Sines & Law of Cosines

2. Use the Law of Sines to find the second angle.



$\frac{\sin F}{14} = \frac{\sin 84°}{15}$

$\sin F = \frac{14\sin 84°}{15} = 0.93$
$F = 68°$

3. Use the Triangle Sum Theorem to find the third angle.

$m\angle D = 180° - (84° + 68°) = 28°$

Name: ______________________________

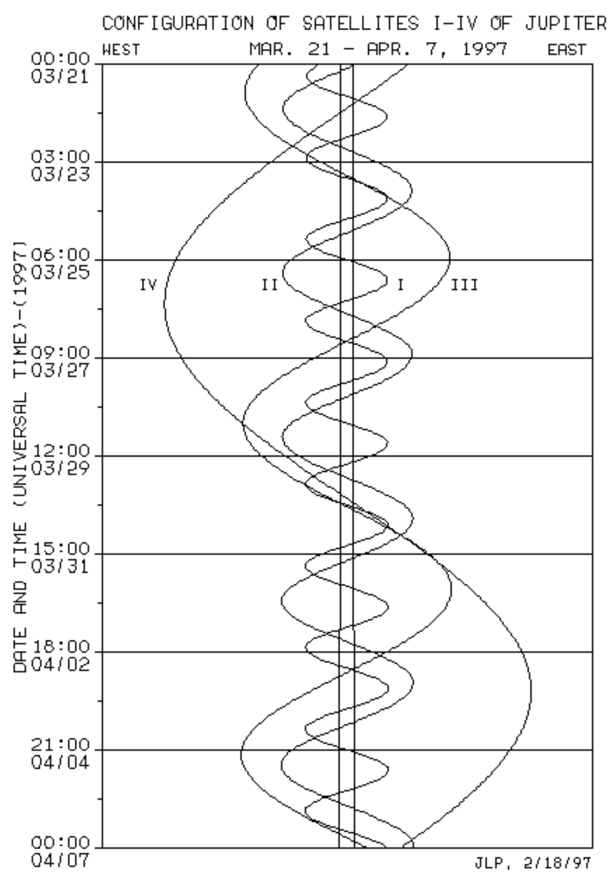
Project: Finding the Moons of Jupiter[9]

1. **Background.** The great astronomer Galileo discovered the largest of Jupiter's moons in 1610. These moons are named:

   I. Io
   II. Europa
   III. Ganymede
   IV. Callisto

   The four moons can easily be observed (as in the picture at right) with a pair of binoculars or a small telescope if one knows where to look for them. In this project, you will draw a number of sketches of the relative positions of Jupiter's moons and gain an understanding of the orbital periods of each.

2. **Task.** Galileo was able to observe Jupiter's four largest moons with his small telescope. When viewed from Earth, they appear to move back and forth in an approximately straight line through the center of the planet.



   In the graph at right, the relative positions of the Galilean moons are depicted for March 21st – Apr 7th, 1997. The narrow band down the center of the graph represents Jupiter while each "wave" is one of the moons, numbered as above.

   a. Using the graph, sketch the planet Jupiter and its four Galilean moons at:

      03:00 UT March 23rd,
      12:00 UT March 29th,
      21:00 UT April 4th

---

[9] Source: Holt, *Geometry*, 2001, p. 653 and http://quest.arc.nasa.gov/galileo/features/findit.html.

Attach your sketches to this document. Be as creative and artistic as you like – I will give extra credit to artistic work.

b. The orbital period of a celestial body such as a moon is the time it takes that body to make one complete orbit of another body. Use the graph to determine the orbital periods of each of the Galilean moons:

I. Io: ____________________

II. Europa: ____________________

III. Ganymede: ____________________

IV. Callisto: ____________________

c. What kind of curve do the lines for the moon's orbits appear to be? Explain why they have this shape. You are encouraged to go online to research the answer to this question but make sure you cite your sources. (Note: the orbits of the four moons are nearly circular.)

____

____

____

____

3. **Grading.** This project count as an assessment and will be graded as follows:

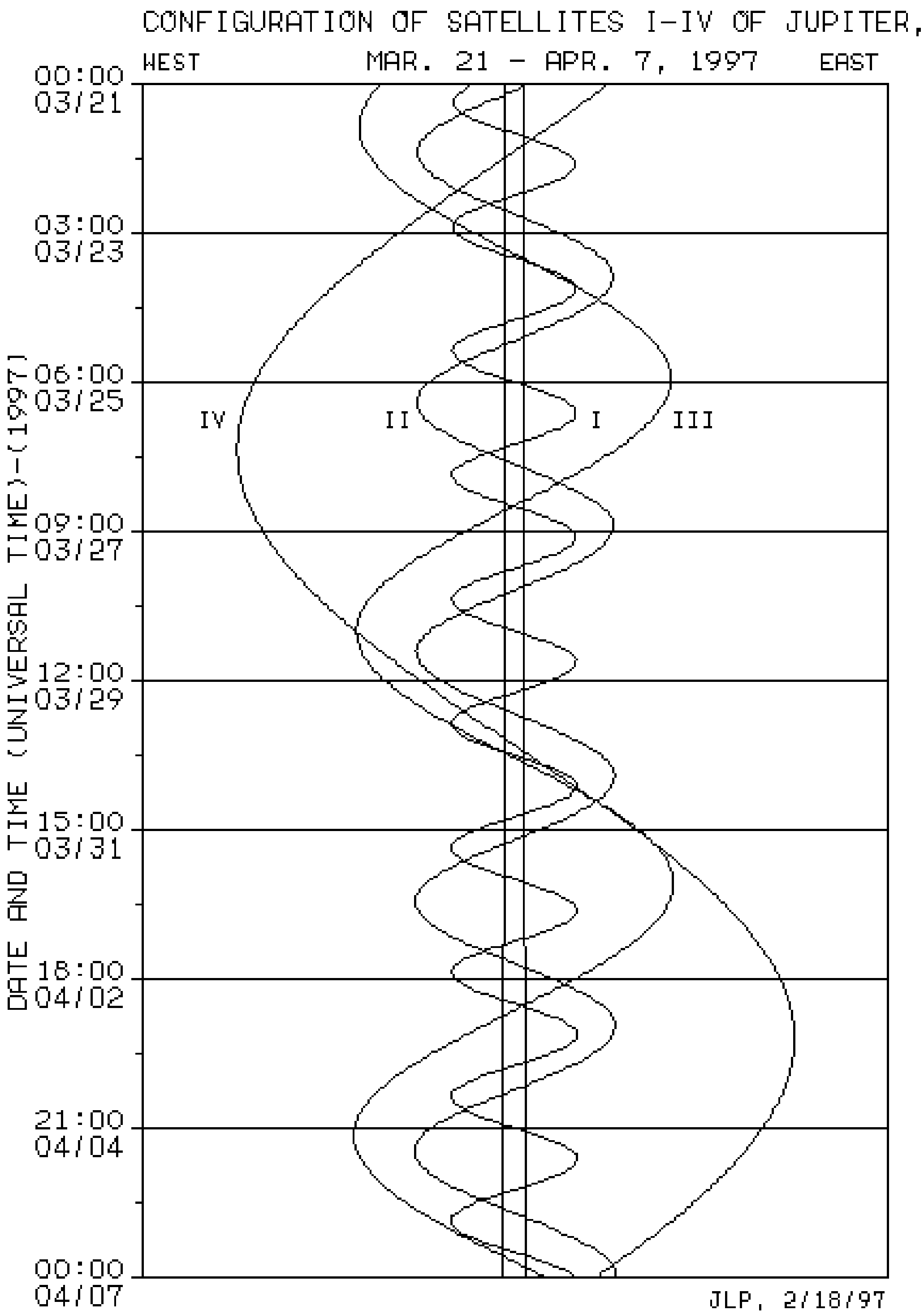
a. Accuracy of sketches: **4 points**

b. Neatness of sketches: **4 points**

c. Artistic quality and creativity of sketches: **4 points**

d. Determination of orbital periods: **4 points**

e. Answer to question about shape of the moons' orbits: **4 points**


CONFIGURATION OF SATELLITES I–IV OF JUPITER,
WEST
MAR. 21 – APR. 7, 1997
EAST
00:00 03/21
03:00 03/23
06:00 03/25
09:00 03/27
12:00 03/29
15:00 03/31
18:00 04/02
21:00 04/04
00:00 04/07
DATE AND TIME (UNIVERSAL TIME)–(1997)
IV
II
I
III
JLP, 2/18/97

Name: ___________________________________

Geometry – February Break Homework

I. **Directions.**

A. This homework will be graded as ***an assessment (in other words, a test)***.

B. Show all work on ***separate sheets of paper stapled to this one.***

C. This assignment is due ***no later than the end of class on Monday February 27th, 2012.***

II. **Circles.**

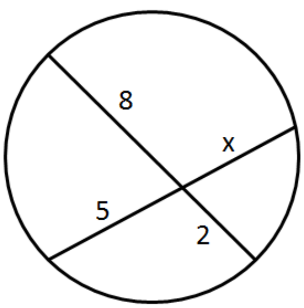
A. Determine the **length** of the arc with the given angle measure.

1. m∠P = 80°; r = 39, ∠P is a central angle.

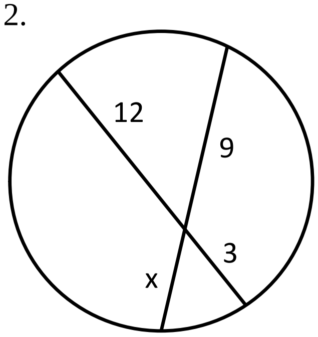
2. m∠P = 82°; r = 5, ∠P is a central angle.

B. Find the value of *x* in each figure.

1.

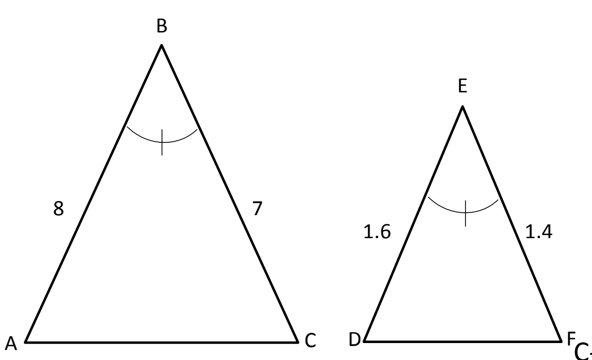


2.

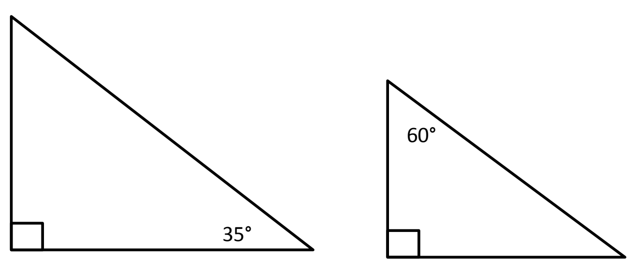


III. **Similarity.** Determine if the pairs of triangles are similar. If so, state which similarity postulate (AA, SSS, SAS) applies.

A.



B.



IV. **Surface Area and Volume of 3D Shapes.**

A. Find the unknown value for a right cylinder with radius *r*, height *h*, and surface area *S*. Round answers to the nearest hundreth.

1. $r = 6,\ h = 3,\ S = ?$

2. $r = 0.5,\ h = 1.2,\ S = ?$

3. $r = 2,\ h = ?\ \ S = 72$

B. Find the surface area and volume of the following:

1. A right cone with $radius = 2.4, height = 5.1, slant\ height = 5.6$.

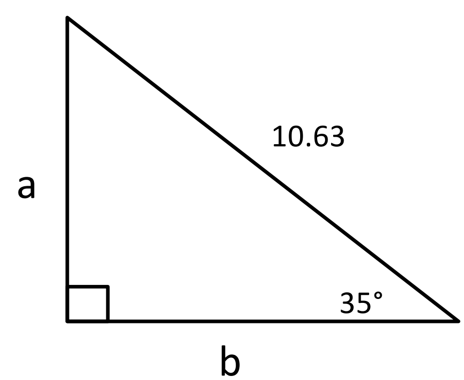
2. A sphere with $radius = 5$.

3. A regular hexagonal prism with $side\ length\ S = 6, apothem\ a = 4, height = 24$.

4. A regular triangular pyramid with $side\ length\ S = 4, slant\ height\ SL = 8$.

V. **Trigonometry.**

A. Using the trigonometric functions we have learned (sine, cosine, tangent) find the lengths of *a* and *b* in the following triangle:



Name: ______________________________

Trigonometry Review I: Circle Graphing and Trigonometry

1. Definitions: Define the following terms.

   a. Trigonometry: ______________________________

   ____

   ____

   b. Tangent: ______________________________

   ____

   ____

   c. Sine: ______________________________

   ____

   ____

   d. Cosine: ______________________________

   ____

   ____

2. Find the *x* and *y* intercepts for the graph of each equation below.

   a. $x^2 + y^2 = 49$

   b. $(x-1)^2 + y^2 = 16$

   c. $(x-3)^2 + (y+4)^2 = 81$

   d. $(x-7)^2 + (y+1)^2 = 121$

3. Find the center and radius of each circle.

a. $(x + 3)^2 + (y - 5)^2 = 36$

b. $x^2 + y^2 = 64$

c. $(x - 3)^2 + (y + 4)^2 = 81$

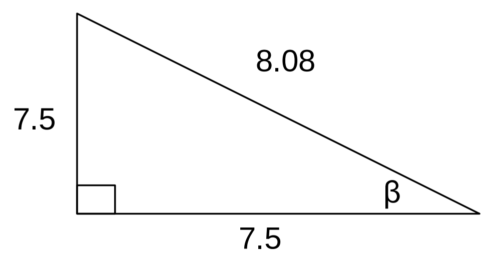
d. $(x - 7)^2 + (y + 1)^2 = 12$

4. Find the sine (sin), cosine (cos) and tangent (tan) for each triangle.

a. sin = ________

cos β = ________

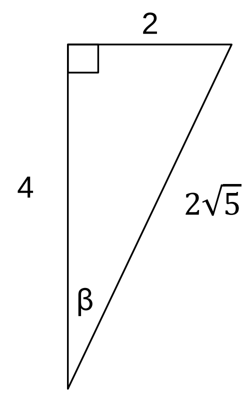
tan β = ________



b. sin β = ________

cos β = ________

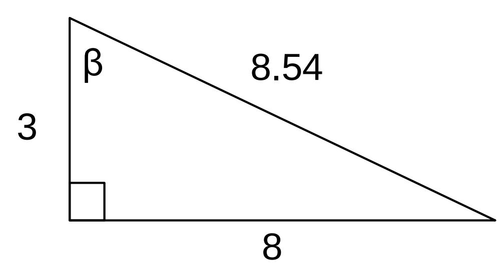
tan β = ________



c. sin β = ________

cos β = ________

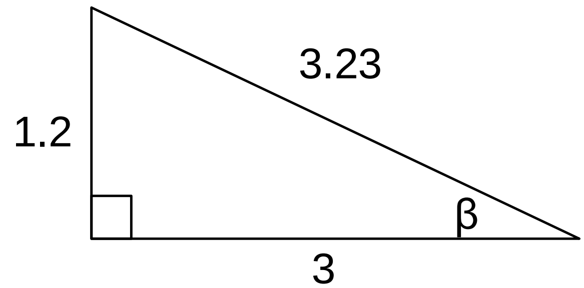
tan β = ________



d. $\sin\beta =$ ________

$\cos\beta =$ ________

$\tan\beta =$ ________



5. Using a calculator, find the following.

a. $\sin 15°$ __________   b. $\cos 47°$ __________   c. $\tan 42°$ __________

d. $\sin^{-1}\frac{2}{\sqrt{13}}$ __________   e. $\cos^{-1}\frac{3}{\sqrt{13}}$ __________   f. $\tan^{-1}\frac{2}{3}$ ___________

6. Extra Credit Challenge Problem. A paraglider is towed behind a boat by 400 ft. ropes attached to the boat at a point 15 feet above the water. The spotter in the boat estimates the angle of the ropes to be 35° above the horizontal.

   a. Draw a sketch of this problem with all known measurements labeled.

   b. Estimate the paraglider's height above the water.

Name: ________________________________

Trigonometry Review II: Circle Graphing and Trigonometry

4. **Vocabulary**. Write the number of the definition on the right next to its term on the left:

h. Trigonometry. ____________	8. On a circle graph, the values of *y* at which *x* equals zero.
i. Tangent. ____________	9. For any point on the circle, the circle can be defined by its radius which is the square root of the sum of the squares of the *x* and *y* coordinates of that point.
j. Sine. ____________	10. The ratio of the leg opposite ∠B to the length of the length adjacent to ∠B in any right triangle having ∠B as one vertex.
k. Cosine. ____________	11. The study of angles and measures in triangles and the relationships between them.
l. *x*-intercepts. ____________	12. On a circle graph, the values of *x* at which *y* equals zero.
m. *y*-intercepts. ____________	13. The ratio of the leg adjacent ∠B to the length of the hypotenuse in a right triangle with ∠B in as one vertex.
n. Circle Equation. ____________	14. The ratio of the leg opposite ∠B to the length of the hypotenuse in a right triangle with ∠B in as one vertex.

2. Find the *x* and *y* intercepts for the graph of each equation below.

a. $x^2 + y^2 = 64$

b. $x^2 + y^2 = 37$

d. $x^2 + (y + 2)^2 = 36$

d. $x^2 + (y - 1)^2 = 27$

3. Find the center and radius of each circle.

a. $x^2 + y^2 = 4$

b. $x^2 + y^2 = 45$

c. $(x + 3)^2 + (y - 5)^2 = 31$

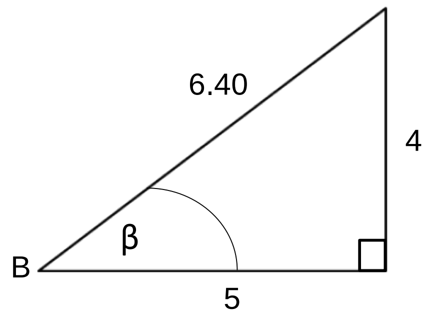
d. $(x - 7)^2 + y^2 = 49$

4. Find the sine (sin), cosine (cos) and tangent (tan) for each triangle.

a. sin β = ________

cos β = ________

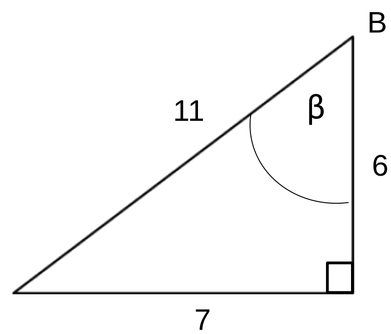
tan β = ________



b. sin β = ________

cos β = ________

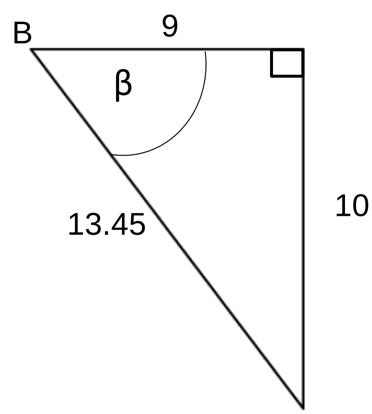
tan β = ________



c. sin β = ________

cos β = ________

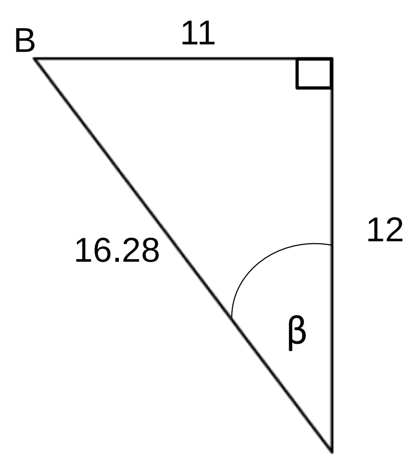
m∠B = ________



d. sin β = ________

tan β = ________

m∠B = ________



5. Using a calculator, find the following.

a. $\sin 25°$ __________ b. $\cos 32°$ __________ c. $\tan 15°$ __________

d. $\sin^{-1} \frac{3}{\sqrt{13}}$ __________ e. $\cos^{-1} \frac{2}{\sqrt{13}}$ __________ f. $\tan^{-1} \frac{3}{2}$ ___________

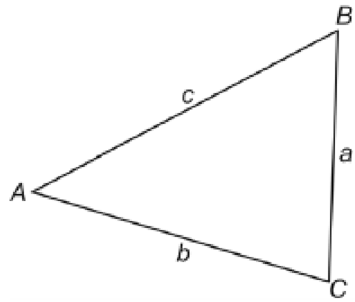
6. Extra Credit Challenge Problem. A spruce tree is approximately the shape of a cone with a slant height of 20 ft. The angle formed by the tree with the ground measures 72°. Estimate the height of the tree rounding to the nearest foot.

The Law of Sines

NAME ___________________________

> Right triangle trigonometry can be used to solve problems involving right triangles. However, many interesting problems involve non-right triangles. In this lesson, you will use right triangle trigonometry to develop the *Law of Sines*. The law of sines is important because it can be used to solve problems involving non-right triangles as well as right triangles.

Consider oblique ΔABC shown to the right.

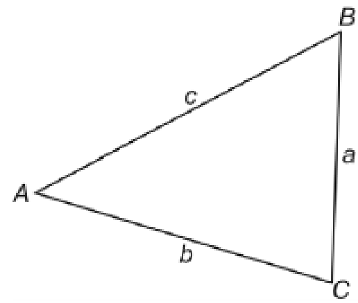


1. Sketch an altitude from vertex B.
2. Label the altitude k.
3. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles, and $\angle C$ is contained in the other. Using right triangle trigonometry, write two equations, one involving $\sin A$, and one involving $\sin C$.

   $\sin A = \frac{\quad}{\quad}$ $\qquad\qquad$ $\sin C = \frac{\quad}{\quad}$

4. Notice that each of the equations in Question 3 involves k. (Why does this happen?) Solve each equation for k.
5. Since both equations in Question 4 are equal to k, they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.
6. Notice that the equation in Question 5 no longer involves k. (Why not?) Write an equation equivalent to the equation in Question 5, regrouping a with $\sin A$ and c with $\sin C$.

Again, consider oblique ΔABC.



7. This time, sketch an altitude from vertex C.

8. Label the altitude k.

9. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles and $\angle B$ is contained in the other. Using right triangle trigonometry, write two equations, one involving sin A and one involving sin B.

$\sin A =$ ______ $\sin B =$ ______

10. Notice that each of the equations in Question 9 involves k. (Why does this happen?) Solve each equation for k.

11. Since both equations in Question 10 are equal to k, they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.

12. Notice that the equation in Question 11 no longer involves k. (Why not?) Write an equation equivalent to the equation in Question 11, regrouping a with sin A and b with sin B.

13. Use the equations in Question 6 and Question 12 to write a third equation involving b, c, sin B, and sin C.

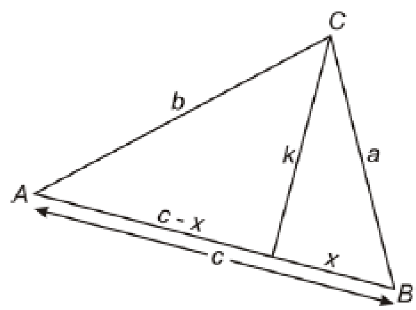
Together, the equations in Questions 6, 12, and 13 form the *Law of Sines*. The law of sines is important, because it can be used to solve problems involving both right and non-right triangles, because it involves only the sides and angles of a triangle.

The Law of Cosines

NAME ____________________

The law of sines can be used to determine the measures of missing angles and sides of triangles when the measures of two angles and a side (AAS or ASA) or the measures of two sides and a non-included angle (SSA) are known. However, the law of sines cannot be used to determine the measures of missing angles and sides of triangles when the measures of two sides and an included angle (SAS) or the measures of three sides (SSS) are known. Since the law of sines can only be used in certain situations, we need to develop another method to address the other possible cases. This new method is called the **Law of Cosines**.

To develop the **law of cosines**, begin with ΔABC. From vertex C, altitude k is drawn and separates side c into segments x and $c - x$. (Why can the segments be represented in this way?)



1. The altitude separates ΔABC into two right triangles. Use the Pythagorean theorem to write two equations, one relating k, b, and $c - x$, and another relating a, k, and x.

2. Notice that both equations contain k^2. (Why?) Solve each equation for k^2.

3. Since both of the equations in Question 2 are equal to k^2, they can be set equal to each other. (Why is this true?) Set the equations equal to each other to form a new equation.

4. Notice that the equation in Question 3 involves x. However, x is not a side of ΔABC. As a result, we will attempt to rewrite the equation in Question 3 so that it does not include x. Begin by expanding the quantity $(c-x)^2$.

5. Solve the equation in Question 4 for b^2.

6. The equation in Question 5 still involves x. To eliminate x from the equation, we will attempt to substitute an equivalent expression for x. Write an equation involving both $\cos B$ and x. (Why use $\cos B$?)

7. Solve the equation from Question 6 for x. (Why solve for x?)

8. Substitute the equivalent expression for x into the equation from Question 5. The resulting equation contains only sides and angles of ΔABC. This equation is called the **Law of Cosines**.

9. Using a similar method, two other forms of this law could be developed for a^2 and c^2. Based on your work for Questions 1–8, write the two other forms of the law of cosines for ΔABC.

Name: ___________________________________

Trigonometry Review III – Laws of Sines and Cosines

I. **Find the indicated measures using the Law of Sines.**

a. $m\angle A = 52°$ $m\angle B = 68°$ $b = 4.2\ cm$ $a =$ _________

b. $m\angle A = 72°$ $m\angle C = 32°$ $a = 1.4\ cm$ $c =$ _________

c. $m\angle B = 64°$ $a = 2.34\ cm$ $b = 3.5\ cm$ $m\angle A =$ _____

II. **Find all unknown sides and angles in the triangle.**

a. $m\angle P = 36°$ $m\angle Q = 68°$ $q = 7$

III. **Find the indicated measures using the Law of Cosines and / or Law of Sines.**

a. $a = 1$ $\quad$ $b = 13$ $\quad$ $c =$ _______ $\quad$ $m\angle C = 20°$

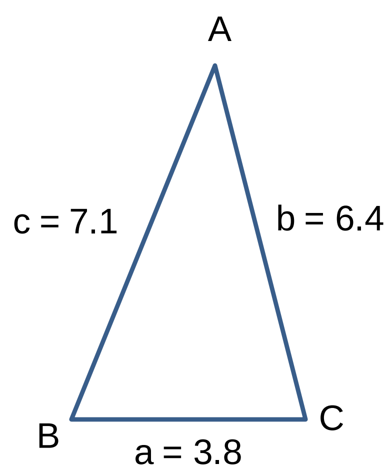
b. $a = 4$ $\quad$ $b = 7$ $\quad$ $c = 5$ $\quad$ $m\angle C =$ _______

IV. **Solve the triangle.**

a. $m\angle A =$ _______________

b. $m\angle B =$ _______________

c. $m\angle C =$ _______________



Name: ___________________________________

Trigonometry Review IV – Laws of Sines and Cosines

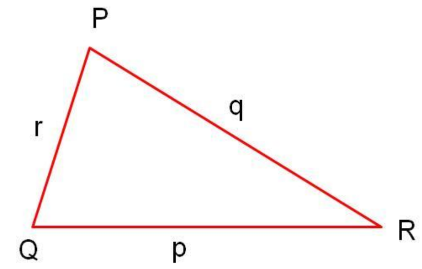
I. **Find the indicated measures using the Law of Sines.**

a. $m\angle A = 25°$ $m\angle B = 65°$ $b = 4.2\ cm$ $a =$ _________

b. $m\angle A = 72°$ $m\angle B = 76°$ $a = 1.4\ cm$ $b =$ _________

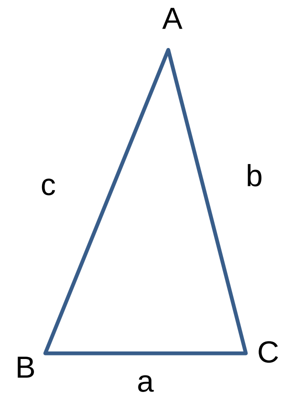
II. **Find all unknown sides and angles in the triangle.**

a. $m\angle R = 75°$ $p = 10$ $r = 14$



b. $m\angle Q = 40°$ $m\angle P = 25°$ $q = 12$

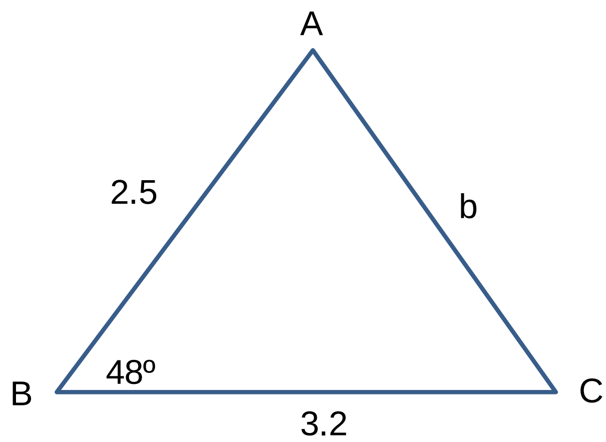
III. **Find the indicated measures using the Law of Cosines and / or Law of Sines.**



a. $a = 12$ $b = 5$ $c =$ _____ $m\angle C = 68°$

b. $a = 9$ $b = 3$ $c = 8$ $m\angle C =$ _______

IV. **Solve the triangle.**



a. $m\angle A =$ _______________

b. $m\angle C =$ _______________

c. $b =$ _______________

V. **Challenge Problem.**

a. Mark and Stephen walk into the woods along lines that form a 72º angle. If Mark walks 2.8 mph and Stephen walks 4.2 mph, how far apart will they be after three hours?